Generalization of 6×6 Magic Square

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Abstract: A magic square of order n is $n \times n$ matrix containing integers in such a way that each row and column add up to the same value. We generalize this notion to that of a 6×6 matrix with the help of a special geometrical figure without having much knowledge of algebra and another branch of mathematics.

Key Words: *Magic Square, Square matrix, Integer, Required sum*

1. Introduction and Review of Literature

A magic square is an $n \times n$ matrix filled with the integers in such a way that the sum of the numbers in each row, each column or diagonally also remain same, in which one integer use at once only. Magic square and related classes of integer matrices have been studied extensively and still it is going on. As we know, ancient India is the origin of numbers and its properties, then after it spreads all over the world. The interested reader may consult (Agnew, 1971, Harold, 1978, Hanson, 1979, Joseph, 1986, Brown, 2001, Ojha & Kaul, 2011 and Ojha & Kaul, 2011). But, the reader can easily extract that there is no requirement of knowledge of algebra, number and its properties and many different branches of mathematics for magic squares. Here, we start with 6×6 matrix with the help of special geometry, which makes us able to obtain a magic square only having the knowledge of sum and multiplication of integers. The sum of six square numbers in all directions can be achieved the equal by using a special figure. Starting a number from one end to the other end clockwise or anticlockwise serially, we can achieve this easily by placing the figure in four different positions. Any number 111 (One hundred eleven) or above divisible by six (6) and with remainder three (3), can be achieved in all directions using each number once only as below:

Example 1: We want the sum one hundred eleven (111). Then divide 111 with 6, we get 18 with remainder 3. Then after subtract 17 a constant number i.e., $18-17=1$. Now, we can start with 1 and taking difference between numbers also 1. If multiples of constant 17 would have been subtracted, the difference between numbers would have been 1,3,5,7 and so on, for the numbers divisible by 6 with remainder 3.

111	111	111	111	111	111	111	111
111	30	29	28	9	8	7	111
111	12	11	10	27	26	25	111
111	18	17	16	24	14	22	111
111	19	20	21	13	23	15	111
111	31	32	33	4	5	6	111
111	1	2	3	34	35	36	111
111	111	111	111	111	111	111	111

2. Methodology and Results

Example 2: We want the sum one hundred seventeen (117). Then divide 117 with 6, we get 19 with remainder 3. Then after subtract 17 a constant number i.e., $19-17=2$. Now, we can start with 2 and taking difference between numbers also 1.

Example 3: We want the sum two hundred seven (207). Then divide 207 with 6, we get 34 with remainder 3. Then after subtract 17 a constant number i.e., $34-17=17$. Now, we can start with 17 and taking difference between numbers also 1.

Example 4: We want the sum two hundred sixteen (216). Then divide 216 with 6, we get 36 without remainder. Then after subtract 35 a constant number i.e., $36-35=1$. Now, we can start with 1 and taking difference between numbers also 2. If multiples of constant 35 would have been subtracted, the difference between numbers would have been 2,4,6,8 and so on, for the numbers divisible by 6 without remainder. Sum Required (SR) =216. $216/6= 36$, $36-35=1$. Then starting number is 1 and the difference between numbers is 2 i.e., $a=1$ and $d=2$.

The table given below shows some examples of starting numbers and the difference kept between them, it is kept according to the subtraction of constants. The numbers divisible by 6 with remainder 3, they should be subtracted by the corresponding constants 17, 52, 87, 122 or addition of 35. If the number is divisible by 6 without any remainder should be subtracted by the constants 35, 70, 105,140 or addition of 35 to get the different combinations of starting numbers and difference kept between them.

Table 1: Examples of Starting Numbers

S. No.	Sum Required (SR)	SR/6	Reminder	Starting number(a)	Difference
1	111	18	3	1	1
2	117	19	3	2	1
3	123	20	3	3	1
4	129	21	3	4	1
5	135	22	3	5	1
6	141	23	3	6	1
7	147	24	3	7	1
8	153	25	3	8	1
9	213	35-17	3	18	1
10	219	36-17	3	19	1

Table 2: Examples of Starting Numbers

S. No.	Sum Required (SR)	SR/6	Reminder	Starting number(a)	Difference
1	213	35-17	3	18	1
2	216	36	0	36-35=1	2
3	219	36-17	3	19	1
4	222	37	0	37-35=2	2
5	225	37	3	37-17=20	1
6	228	38	0	38-35=3	2
7	333	55	3	55-52=3	3
				55-17=38	1
8	336	56	0	56-35=21	2
9	426	71	0	71-35=36	2
				71-70=1	4
10	531	88	3	88-17=71	1
				88-52=36	3
				88-87=1	5
11	636	106	0	106-35=71	2
				106-70=36	4
				106-105=1	6

Example 6: (S.No.4, Table 2): SR=222, a = 2, d = 2, reminder(R) = 0.

222	222	222	222	222	222	222	222	222
222	60	58	56	18	16	14	222	222
222	24	22	20	54	52	50	222	222
222	36	34	32	48	46	44	222	222
222	38	40	42	26	46	30	222	222
222	62	64	66	8	10	12	222	222
222	2	4	6	68	70	72	222	222
222	222	222	222	222	222	222	222	222

Example 7: (S.No.9, Table 2): SR=426, a = 36, d = 2, reminder(R) = 0.

SR=426, a = 1, d =4, reminder(R) = 0

An equation has been formed as $2a + 35d = \frac{SR}{3}$, where a is starting number, d is difference between numbers

used and SR is sum required. Let SR = 426, $2a + 35d = \frac{426}{3} = 142$. If d=2, then a = 36. If d=4, then a=1

Example 8: (S.No.10, Table 2): SR=531, a= 71, d=1, Reminder =3.

SR=531, a= 36, d=3, Reminder =3.

3. Our Process

Steps: Fix the required total sum ($111 \leq S < \infty$), Then there exist two favorable cases (i) $S = 6p, p \in I$ (ii) $S = 6p+3, p \in I$. In case (i) Now, We have to decide starting number $n_1 = p - [w = \{\text{Number of blocks} - 1\}]$. In case (i) Now, We have to decide starting number $n_1 = p - [w = \{\frac{\text{Number of blocks}}{2} - 1\}]$. Then calculate the sixteen numbers $n_2 = n_1 + d$, where d is predefined by problem, if not then take. $d = 1, n_1, n_2, n_3, \dots, n_{16}$. Later on W may be change after the fixed limit, but it will change in the manner $W = nw + 1, n \in I$. Then arrange these sixteen numbers with the help of suggested geometrical figure. We can find numerous solutions with the help of rotation of suggested figure in clockwise and anticlockwise direction. We can find numerous solutions by defining the suitable d . But in all cases, we find optimized sum required. In case (ii), only d will be even with the same process.

Properties of our Process: We may obtain various starting number for fix required sum. We may obtain various starting number with different difference for fixed required sum. We may get difference between numbers variable. We may get different matrix with same set of numbers, with same difference via rotating the direction of figure from starting to end point in clock wise or anti clock wise direction and also can change the starting and end point for getting different matrix. So, we can start for finding magic square either with required sum or starting point or with difference or with rotation of figure. Finally, we are working on 7×7 magic square matrix on the same line. Further we will explore some more properties of this geometric figure in our subsequent article.

Discussion on Applications: Magic squares are useful in cryptography for coding and decoding the message in secured manner with complete integrity. The concept of balancing the magical squares with the help of suggested geometrical figure is easy and perfect. This is a new concept for magical squares. It is being used for moving bodies, Rotors, Cam shaft, Crank shaft etc.

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