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Generalization of 5×5 Magic Square

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Abstract: A magic square of order n is n×n matrix containing integers in such a way that each row and column add up to the same value. We generalize this notion to that of a 5×5 matrix with the help of a special geometrical figure without having much knowledge of algebra and another branch of mathematics. As we know, from very beginning common man always attracts with square magic properties and always tries through various ways to achieve it quickly. Here, we are giving effortless way to achieve it.

Key Words: *Magic Square, Square matrix, Integer, Required sum*

1. Introduction

A magic square is $n \times n$ matrix filled with the integers in such a way that the some of the numbers in each row, each Coolum or diagonally remain the same, in which one integer used once only. Magic square and related class of integers of matrices have been studied carefully and still going on. As it is known fact that India is the origin of numbers and its properties, which spread all over the world. We start with 5x5 matrix by using a specially developed geometrical figure which can be put on 5x5 squares and starting with a number from one end to other end (clockwise and anti clockwise) serially. We can easily achieve by placing the figure in four different positions (Clockwise or anticlockwise). Any number sixty five (65) or above to infinity divisible by 5, the required sum can be easily achieved the same in all direction by using 25 numbers serially using only once.

In recreational mathematics, a magic square of order n is an arrangement of n^2 numbers, usually distinct integers, in a square, such that the sum of n numbers in all rows, all columns and both diagonals is the same constant called the magic number. The magic square in normal is represented using n x n matrix. A normal magic square contains the integers from 1 to n^2. Normal magic squares exist for all orders $n \geq 1$, except n = 2. The magic constant for normal magic squares of order n is given by n(n2 + 1)/2. There are several methods for constructing the magic square of any given order. This paper proposes algorithms to obtain the magic square of any given order n. Some of the algorithms are straight forward and others are designed using the divide and conquer technique. The magic squares have several applications in fields of discrete and combinatorial mathematics, and also in the area of graph theory. One such application of magic square is in graph labeling. It has been proved using the magic square of order n that, "There exists a vertex magic total labeling for all complete graph K_n" (Krishnappa et al., 2009, 2010). If the developed figure is placed on 5×5 matrix and starting a number serially from one end to the other end, we can get the sum of each line same as below:-

65	**65**	**65**	**65**	**65**	**65**	**65**
65	**9**	**10**	**22**	**23**	**1**	**65**
65	**8**	**21**	**20**	**2**	**14**	**65**
65	**11**	**7**	**13**	**19**	**15**	**65**
65	**12**	**24**	**6**	**5**	**18**	**65**
65	**25**	**3**	**4**	**16**	**17**	**65**
65	**65**	**65**	**65**	**65**	**65**	**65**

200	**200**	**200**	**200**	**200**	**200**	**200**
200	**76**	**37**	**34**	**25**	**28**	**200**
200	**13**	**73**	**22**	**61**	**31**	**200**
200	**10**	**16**	**40**	**64**	**70**	**200**
200	**49**	**19**	**58**	**7**	**67**	**200**
200	**52**	**55**	**46**	**43**	**4**	**200**
200	**200**	**200**	**200**	**200**	**200**	**200**

Starting by the number got by dividing the required hole sum numbers by five (5) and subtracting it by a constant twelve(12)or multiples of 12, the number got will be the starting number such as we want the sum of five numbers in all directions should be sixty five (65), divided by five $65 \div 5 = 13$. Subtract a content number 12 from 13 we get 1, we can start from 1, from one end of figure clockwise or anticlockwise using 25 numbers serially to get the sum of all deferent numbers of five squares as 65 in all directions.

2. Main Result

A figure has been developed, if this figure is placed on 5×5 matrix square block clockwise or anticlockwise the sum of all the line in all direction can be the same. Figure given below :-

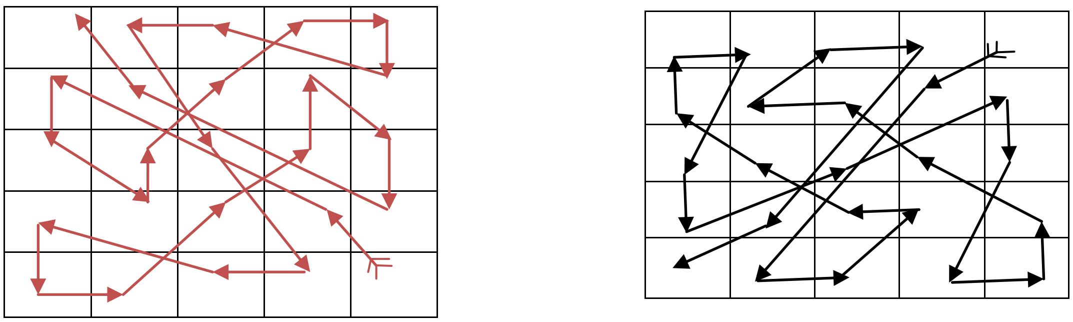


Figure: - Image showing figure developed with 25 square blocks.

Any numbers sixty five (65) or above divisible by five can be got in all the directions by putting the above figure in different positions. For example, we want 425 four twenty five the sum of all the numbers in all the directions of 5x5 square block. We must first divide it with five as (425÷5=85) and then subtract 12 from it. i.e. (85-12=73). We can start with 73 seventy three in arithmetical progression that is 73,74,75 and so on to get 425 sum in all directions as below:-

97	84	83	80	81	425
76	96	79	92	82	425
75	76	85	93	95	425
88	78	91	74	94	425
89	90	87	86	73	425
425	425	425	425	425	425

The same method can be used with other numbers having different equal difference bet been than like numbers:-

$1, 2, 3, 4$ ……………… Arithmetical progression

$$\left.\begin{array}{l} 1, 3, 5, 7 \ldots\ldots\ldots\ldots\ldots\ldots \\ 1, 4, 7, 10 \ldots\ldots\ldots\ldots\ldots\ldots \\ 1, 5, 9, 13, \ldots\ldots\ldots\ldots\ldots\ldots \end{array}\right\} \quad \textit{Geometrical progression}$$

The above four hundred twenty five (425) sum can be archive with using other numbers also, having different equal difference, but subtracting factor will be multiples of constant twelve. Suppose we want the sum of two hundred and forty five (245) on all sides of the 5×5 square. The following combinations are possible to get the sum of two hundred and forty five (245) on all sides, by dividing the sum with 5 we get $245 \div 5 = 49$

After Subtracting Constant	Starting Number	Difference between numbers
$245 \div 5 = 49 - 1\ 12\ = 37$	37	1
$49 - 2\ 12\ = 25$	25	2
$49 - 3\ 12\ = 13$	13	3
$49 - 4\ 12\ = 1$	1	4

245	245	245	245	245	245	245
245	61	48	47	44	45	245
245	40	60	43	56	46	245
245	39	41	49	57	59	245
245	52	42	55	38	58	245
245	53	54	51	50	37	245
245	245	245	245	245	245	245

245	245	245	245	245	245	245
245	73	47	45	39	41	245
245	31	71	37	63	43	245
245	29	33	49	65	69	245
245	55	35	61	27	67	245
245	57	59	53	51	25	245
245	245	245	245	245	245	245

245	245	245	245	245	245	245
245	85	46	43	34	37	245
245	22	82	31	70	40	245
245	19	25	49	73	79	245
245	58	28	67	16	76	245
245	61	64	55	52	13	245
245	245	245	245	245	245	245

245	245	245	245	245	245	245
245	97	45	41	29	33	245
245	13	93	25	77	37	245
245	9	17	49	81	89	245
245	61	21	73	5	85	245
245	65	69	57	53	1	245
245	245	245	245	245	245	245

For the required sum of two hundred and forty five can have Twelve (12) more sequences by rotating the figure, so total sixteen (16) combinations / sequences are possible to get the required sum of two hundred and forty five(245). The higher number has higher combinations possible.

3. Discussion

In case of 5×5 Twenty five square block a method may be adopted to get the same sum of all the directions. First of all the number produced after dividing the required sum with five(5) should be kept in the center of the block and subtracting twelve of multiple of twelve from it to get the stating number and difference between the numbers and then place the figure in any direction. Now start with stating number and using twenty five numbers with keeping the procured difference between them, we will get the required sum in all directions. An equation has been developed to get starting number and difference between numbers used easily. Since the total sum of the twenty five numbers used in any combinations for the same given required sum remains the same and is five times of the given required sum, taking the advantage of this, a formula has been developed

$$a + 12d = \frac{SR}{5}$$

Where a = Starting number
d = Difference between numbers
SR = Sum required

Example 1:- Let the number required is one hundred and eighty five (185).

Then $a+12d=\frac{SR}{5}$, sum required(SR)= 185 let *d=1* .

Hence, $a+12d=\frac{185}{5}=37 \Rightarrow a=37-12=25$, .

49	36	35	32	33	185
28	48	31	44	34	185
27	29	37	45	47	185
40	30	43	26	46	185
41	42	39	38	25	185
185	185	185	185	185	185

Again, let $d=2$,
$a+12d=\frac{185}{5}=37 \Rightarrow a=37-24=13$.

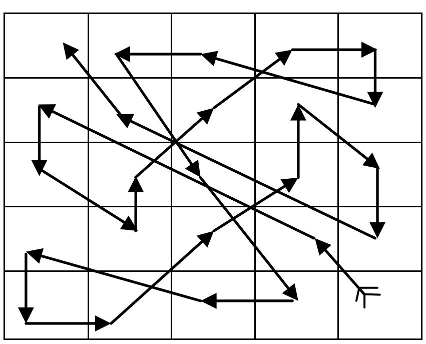
61	35	33	27	29	185
19	59	25	51	31	185
17	21	37	53	57	185
43	23	49	15	55	185
45	47	41	39	13	185
185	185	185	185	185	185

Again, let $d=3$,

$a+12d=\frac{185}{5}=37 \Rightarrow a=37-36=1$.

73	34	31	22	25	185
10	70	19	58	28	185
7	13	37	61	67	185
46	16	55	4	64	185
49	52	43	40	1	185
185	185	185	185	185	185

The above three combinations will have three more combinations/ sequences each by rotating the figure in other directions, that means, we have twelve combinations to have 185 in all directions. The other method also can be adopted to get the first number and difference between the numbers .By above method, we can be able to get any number divisible by five not lower than sixty five(65) to infinity, achieved in *5x5*.



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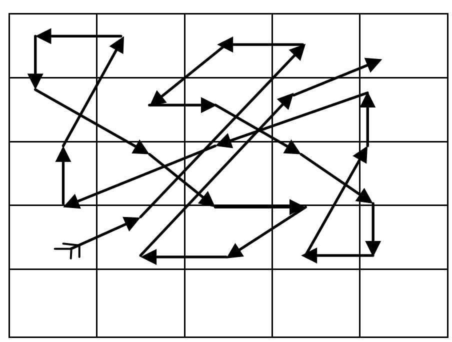
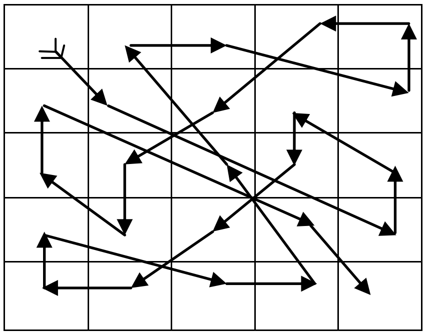
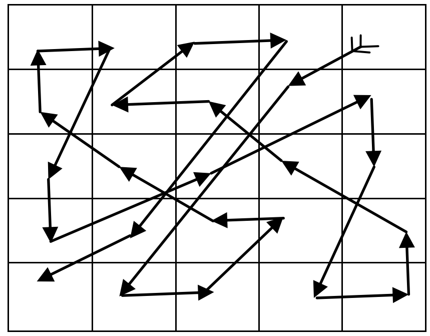


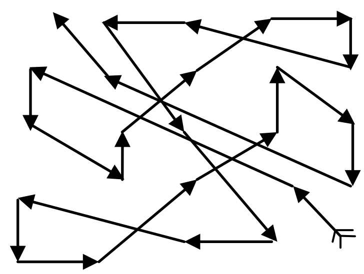




4. Our Process

Steps:

- Fix the required total sum($65 \le S < \infty$), Then there exist two favourable cases $S = 5p, p \in I$.
- Now, We have to decide starting number $n_1 = p - [w = \{\frac{\text{Number of blocks+1}}{2} - 1\}]$.
- Then calculate the sixteen numbers $n_2 = n_1 + d$, where d is predefined by problem, if not then take $d = 1$, $n_1, n_2, n_3, .,.,..n_{16}$. Later on w may be change after the fixed limit, but it will change in the manner $W = nw + 1, n \in I$.
- Then arrange these twenty five numbers with the help of suggested geometrical figure.
- We can find numerous solution with the help of rotation of suggested figure in clockwise and anticlockwise direction.
- We can find numerous solution by defining the suitable d.
- But in all cases, we find optimized sum required.
- Required figure



5. Properties of our Process

- We may obtain various strating number for fix required sum
- We may obtain various strating number with different difference for fixed required sum
- We may get difference beetween numbers variable
- We may get different matrix with same set of numbers, with same differnce via rotating the direction of figure from strating to end point in clock wise or anit clock wise direction and also can change the strating and end point for getting different matrix
- So, we can start for finding magic square either with required sum or with starting point, with difference, or with rotation of figure.

Finally, we are working on 6×6 magic square matrix on the same line. Further, we will explore some more properties of this geometric figure in our subsequent article.

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