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**Consumption with Imperfect Income Expectations**

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**Abstract:** Using survey forecast data, this paper documents several stylized facts about forecasters' beliefs on income and consumption and aggregate consumption growth: (1) survey-based income forecast at consensus level is highly correlated with consumption growth; (2) consensus income and consumption growth forecast errors under-react to macro news shocks and (3) consensus income forecast error and consumption growth under-react initially and overreact subsequently in response to main business cycle shocks. Motivated by this evidence, we propose a model of equilibrium consumption determination where agents learn the exogenous latent permanent income process and extrapolate the past income realizations. Our model can generate the behavior of consumption that the rational-expectation Permanent Income Hypothesis fails to predict excess smoothness and excess sensitivity of aggregate consumption, and negatively correlated consumption growth and past income change in the medium run. This study contributes to the literature on economic belief formation and empirical consumption by showing how survey-motivated evidence can jointly explain a range of important anomalies.

**Keywords:** *Consumption, Income, growth, business cycle shocks*

## 1. Introduction

The rational-expectation Permanent Income Hypothesis (PIH - hereafter) predicts that consumers with a concave utility function spread out income shocks to smooth consumption and that lifetime consumption, instead of current income, determines consumers' optimal consumption level.[1] Moreover, when there is uncertainty about future income, the consumption Euler equation reduces to a certainty equivalence condition when the marginal utility is a linear function in consumption. It can also be shown that the optimal consumption level is a martingale and that consumption growth is only a function of the prediction error for permanent income, which is orthogonal to any past information. However, empirical studies of consumption have found some patterns in aggregate consumption that are not consistent with predictions from PIH. For example, the seminal papers by Flavin (1981), Deaton (1986) and Campbell & Deaton (1989) use U.S. data to show that aggregate consumption is excessively smooth in that it responds too little to unpredictable changes (innovations) in income and is excessively sensitive in that it responds too much to predictable changes (realized changes) in income.

Campbell & Deaton (1989) argue that these two observations are intrinsically related and conclude that consumption underreacts to permanent income shocks and thus adjusts with a delay. Furthermore, Beeler & Campbell (2012) show that consumption growth and past income change are negatively correlated in the medium term, which also contradicts the prediction from the PIH. In addition to the empirical observations regarding the relation between income changes and aggregate consumption growth, consumers' lifecycle consumption profile in the data seems inconsistent with a standard consumption-saving theory based on PIH. Specifically, consumption initially grows during the lifecycle and drops toward the end of life (Fernandez-Villaverde & Krueger, 2007 and Gourinchas & Parker, 2003), which both use the Consumer Expenditure Survey data and document a hump-shaped lifecycle consumption profile). This paper aims to bring new evidence from survey data to elucidate the aforementioned consumption anomalies. Specifically, we combine the Survey of Professional Forecasters data with business cycle news shocks to study biases in the quarterly income (proxied by Gross Domestic Product) forecast panel by estimating the impulse response function of the forecast errors in response to news shocks.[2]

[1] The reason is that consumers want to smooth their consumption flow under the assumption of concave utility. The optimal consumption mainly depends on the average level of lifetime income because it is much smoother than the income in any particular period.

[2] As suggested by (Kucinskas & Peters, 2022), these biases can be inferred from the response of forecast errors to past news by flexibly estimating the impulse response function of forecast errors without precise knowledge of the true data-generating process.

We show that annual consumption growth is predictable from the average forecast errors reported by the panel of forecasters and consumption growth's impulse response function has a similar shape to that of the consensus forecast errors. We find that income forecast errors underreact or overreact to different pieces of news at multiple horizons, resulting in forecast errors being predicted by news innovations. This is further reflected in the observed consumption growth path which itself is predictable from past information about income. Using survey data, we document several interesting empirical facts about income forecasts, consumption forecasts and observed consumption growth. The first piece of evidence we show is that we cannot reject the certainty equivalence condition by using survey-based expectations. Although testing the validity of an Euler equation is much more challenging and is beyond the scope of this paper, we see from this result that survey-based expectations can be useful. The fact that market participants report forecasts of income that are much closer to the PIH is important. First, it may suggest that incorrect or non-rational beliefs may play an important role in explaining the failure of the PIH, which may help explain some consumption anomalies. This, in fact, is the main story we want to convey in this paper: Survey-based beliefs capture some systematic expectational errors, which makes consumption growth predictable. Furthermore, positing that the PIH Euler equation holds if we incorporate survey-based beliefs provides a useful benchmark as a stylized observation to support the starting point of our theory.

The second empirical observation we document is that the consensus income forecast errors are highly and positively correlated with contemporaneous consumption growth with a 1-, 2-, 3- and 4-quarter looking-back window. The rational PIH framework predicts that the forecast error of the permanent income component is a sufficient statistic for aggregate consumption growth. The second empirical observation is consistent with this prediction. However, we do not argue that the income forecast error is the only predictor of the consumption growth process. Instead, we see, again, that survey-based forecasts can be quite useful and may play an important role in explaining variation in the aggregate consumption path. These two empirical facts imply that survey-based expectations of income may help explain certain deviations from the random walk assumption of consumption growth. Therefore, we carefully study the income belief formation process and especially the biases implied in the survey forecast data. The third empirical observation replicates the results by (Bordalo, Gennaioli, Ma, & Shleifer, 2020) using an updated data sample and suggests that individuals consistently overreact to new pieces of information when they forecast income and consumption. For both nominal and real income, the consensus-level forecasts underreact to news such that forecasters do not fully adjust their forecasts when receiving new information about the data. The evidence that forecasters underreact to the news at the average level is important and a key piece of evidence we use to explain the excess smoothness puzzle. Forecasters are sluggish in incorporating new information about income into aggregate consumption forecasts, which causes consumption forecasts to under-respond to income innovations.

We rely on a local projection regression method and determine the impulse response function of the consensus level forecast errors of income. This exercise shows that after the arrival of a shock, the consensus forecasts of real or nominal income are persistently lower than the realized values, indicating an underreaction to the news. However, after this initial response, forecasted income exceeds realized income, indicating a subsequent overreaction of the forecasts. These patterns are captured by initial positive forecast errors, followed by negative forecast errors. Similar behaviors have been documented for inflation/unemployment forecasts (Angeletos, Huo, & Sastry, 2020) and for U.S. interest rate/interest rate differential forecasts (Vasudevan, Valente, & Wu, 2022). Using realized annual consumption growth data and plotting its impulse response function, we observe a similar pattern of initial underreaction and delayed overreaction to news shocks. To explain the consumption puzzles in a manner consistent with the survey-based evidence, we construct a simple lifecycle consumption model with frictions. The agents in the model face an exogenously defined and identical income process that we call a fundamental or permanent income process, which statistically is represented by a simple AR (1) process. Agents do not directly observe this underlying latent variable; however, they are endowed with a noisy signal which adds additional transitory shocks to the fundamental variable.[3] We interpret this noisy signal as individual income in each period; we will use the terms" signal" and" individual income" interchangeably in this paper.

[3] We call it "permanent" because the realized income in each period will not die out and spans the course of the lifecycle. In other words, the shocks in this process are permanent instead of transitory.

This is one key friction in our model: Consumers have to learn the underlying data-generating process and forecast future income since they are forward-looking. The other important ingredient our theory uses to generate the observed under-reaction and overreaction pattern is extrapolative beliefs. Agents in our setting do not know the true structural parameter of the permanent income process; rather, they use a subjective persistence coefficient when making forecasts. Our characterization shows that when agents over-extrapolate, the individual-level forecast error overreacts and equilibrium consumption growth at the aggregate level is negatively correlated with past income change in the medium term.[4] The proposed model is able to generate an impulse response function of income expectation that is consistent with the survey-based IRF.

## 2. Related Literature

Our paper is related to several streams of literature and builds on prior studies of consumption behavior and its relation to income changes. An early seminal paper (Caballero, 1990)reconciles three different features of consumption: excess growth, excess smoothness, and excess sensitivity in a framework with precautionary savings. A number of recent papers have proposed several explanations for the puzzling consumption behavior: information costs (Reis, 2006), memory constraints (da Silveira & Woodford, 2019), sticky beliefs (Carroll, Crawley, Slacalek, Tokuoka, & White), self-attribution bias (Zinn, 2013), the spirit of capitalist (Huang & Caliendo, 2011), moral hazard (Attanasio & Pavoni, 2011), habit formation (Chetty & Szeidl, 2016) and loss aversion and reference-dependent preferences (Pagel, 2017). Our paper differs from these prior studies in several ways. First, while most of the aforementioned studies rely on a non-standard preference approach, such as habit formation and costly information processing, the current paper focuses on a belief-based explanation. We complement the existing literature by providing an alternative way to look at the behavior of consumption. Second, we directly use survey-based evidence to inform a possible model of consumption. Similarly, (Vasudevan, Valente, & Wu, 2022) developed a theory of exchange rate determination by studying the behavior of exchange rates using short-term interest rates and interest rate differential forecasts.

Other related evidence, especially about how consensus-level and individual-level expectations react to news shocks, includes (Angeletos, Huo, & Sastry, 2020) which studies inflation and unemployment forecasts from the same data source as this paper, and (Bordalo, Gennaioli, Ma, & Shleifer, 2020) which reports the predictability of forecast errors for a range of macroeconomic and financial aggregate variables. The key finding, that expectations and observed prices on average initially under-react and subsequently overreact to news, has been extensively studied in stock market settings (see, for example, (Bondt & Thaler, 1985 and Cutler, Poterba & Summers, 1991), among others). Our paper complements the existing literature by providing direct evidence concerning income forecasts and aggregate consumption growth. The simple theoretical model we propose in this paper relies on two key ingredients to explain consumption behavior: informational frictions and extrapolation. Prior studies of informational frictions, such as high-order uncertainty, rational inattention and incomplete information, include Sims (2003), Sims (2010), Matejka (2016), Nimark (2008), Berrada (2006) and Timmermann (2001). Our approach to modeling informational friction is straightforward; agents in our model rely on Bayesian learning to extract fundamental information.

Extrapolation plays another central role in our theory. In the literature, extrapolative beliefs have been extensively studied both theoretically (Barberis, Greenwood & Shleifer, 2015) and empirically (Liao, Peng, & Zhu, 2021). Our model of extrapolation is intuitive and inspired by the work of (Angeletos, Huo, & Sastry, 2020). Finally, this paper is broadly connected to the literature on how beliefs, especially incorrect beliefs, affect equilibrium prices. Related papers include, for example, (Gourinchas & Tornell, 2004), who explain time-series variation in spot exchange rates by incorporating a specific form of interest rate belief distortion, and (Barberis, Greenwood, Jin, & Shleifer, 2018), who develop a model with extrapolative beliefs to explain price bubbles and trading volume. Our paper similarly focuses on certain deviations from rational expectations and delivers theoretical predictions that are consistent with the observed consumption growth data. The rest of the paper is organized as follows. The next section discusses the existing empirical evidence on consumption behavior. Section 3 presents the data used in this paper and documents several stylized facts from the survey data. Section 4 and Section 5 propose and characterize a model of consumption under incomplete information and extrapolative beliefs. Finally, Section 6 concludes and suggests some future research directions.

[4] Here over-extrapolation means that agents perceive an even more persistent process when updating their beliefs.

**Puzzling Facts about Consumption:** In this section, we first review the standard PIH theory and its predictions about consumption growth. The core PIH prediction is that the Euler equation from an optimal consumption-savings problem is consistent with a certainty equivalence condition and that consumption change is only a function of the forecast error of lifetime permanent income, i.e. a random walk process. This property makes the consumption growth process a pure innovation that cannot be predicted from any realized past information. However, this conclusion is not supported by the data and there is strong evidence in the literature showing deviations from the PIH, which argues against the unpredictability of consumption growth. Specifically, in contrast to predictions from the PIH theory, aggregate consumption responds too much to past income change and too little to contemporaneous income news. In addition, the PIH also fails to predict the empirical findings that the consumption growth is negatively autocorrelated in the medium term and that the consumption profile over the lifecycle is hump shaped. We will review these findings later in this section. Consider a simple problem in which an agent makes an optimal consumption-saving plan subject to an intertemporal budget constraint.

If we assume that the constant interest rate and the intertemporal discount rate are identical, we can derive the following well-known Euler equation,
$u'(c_t) = \mathrm{E}t(u'(c_{t+1}))$,
where *it* represents the consumption level at time $t$. Furthermore, under quadratic utility, we can show that certain equivalence holds:
$c_t = \mathrm{E}t(c_{t+1})$.
Alternatively, under these assumptions, the Euler equation can also be written as
$ct = ct-1 + \varepsilon t$
where $\mathrm{E}t(\varepsilon_t) = 0$ and $\varepsilon_t$ i.i.d. across time. The economic interpretation of $\varepsilon_t$ is that it represents a consumption innovation that is revealed at time $t$, for example, shocks to personal income at time $t$. An implication of this refined Euler equation is that: consumption growth $\Delta c_t$ is only correlated with the innovation at time $t$ and is orthogonal to any information set before time $t$, and thus $\Delta c_t$ cannot be predicted by past consumption or labor income change. In other words, consumption growth over time should be a random walk. However, numerous studies have rejected this random walk conclusion and have documented the following well-known puzzling empirical facts.[5]

- *Excess sensitivity of consumption growth*. Aggregate consumption responds too much to lagged income changes or to predictable changes in income,
- *Excess smoothness of consumption growth*. Aggregate consumption responds too little to contemporaneous income news or to unpredictable changes in income,
- *Negatively correlated consumption growth over the medium term*. Consumption growth exhibits mean reversion and the autocorrelation of consumption growth is negative,
- *Hump-shaped consumption profile over the lifecycle*. Consumption grows initially and then decreases over the course of the lifecycle, even after controlling for many factors including family size and time.

The PIH is inconsistent with the above empirical findings because the PIH says that consumption growth only responds one-to-one to a current income innovation but not to any past realized income innovations. Furthermore, the PIH does not predict a particular shape of the consumption profile. These pieces of evidence challenge the PIH theory. This paper proposes an alternative theory based on a behavioral mechanism to both explain these puzzling facts and generate new testable hypotheses.

**Evidence from Survey Data:** We begin by presenting several stylized facts using the survey data on income and consumption forecasts both at the aggregate and at the individual level. Some of the evidence has previously been documented in, for example, (Bordalo, Gennaioli, Ma, & Shleifer, 2020). All of the observations exhibit some deviations from the rational PIH model, and they motivate our model assumptions which we present in the next section.

**Data:** The two main datasets we use are the Survey of Professional Forecasters (SPF, hereafter) and the main business cycle shock derived from (Angeletos, Huo, & Sastry, 2020). The SPF data dates back to 1968 and was originally collected by the American Statistical Association and the National Bureau of Economic Research.

[5] See (Jappelli & Pistaferri, 2010) for a more comprehensive survey.

Since 1990, it has been conducted by The Federal Reserve Bank of Philadelphia. The survey is conducted every quarter and contains point forecasts of major macroeconomic and financial variables in the United States. The forecasters in the survey are identified anonymously by forecaster IDs and we observe the forecaster-level data from which we can also construct different forecast moments such as consensus-level (mean or median) forecasts. In each quarter $t$, before the release of the current quarter's realized value, forecasters are asked to provide their estimates for the current quarter and each of the next four quarters. Therefore, in each given quarter $t$, we observe five forecasts for each following quarter. This paper mainly uses three forecast series from the SPF data: Nominal GDP (NGDP), Real GDP (RGDP) and Real Personal Consumption (RCONSUM).[6] The data cleaning procedure follows (Bordalo, Gennaioli, Ma, & Shleifer, 2020).[7] The forecasted variables are in levels and we transform them into implied growth rate forecasts.

Specifically, in a given quarter $t$, we define individual $i$'s annual growth rate forecast as $F_{i,t-1}(x_{t+3})/x_{t-1} - 1$ where $x$ is the macro series of interest. The realized growth rate is constructed in the same fashion, $x_{t+3}/x_{t-1} - 1$. We also define the forecast revision as the revision of the implied growth rate from the last quarter (quarter $t-1$ to quarter $t$), which reflects the change in the forecaster's information set. Finally, the forecast error is defined as the discrepancy between the actual realized value and the forecast. From the individual-level forecasts, we calculate the consensus-level forecast as the mean forecast for each quarter.[8] The second dataset we use is the business cycle shocks. We mainly use these shocks to construct the IRFs of the forecast and forecast errors. These shocks are derived by estimating a joint VaR model and maximizing the contributions to the business cycle variations of several major business cycle variables including unemployment, hours worked, output, consumption, investment and inflation. A more comprehensive description can be found in (Angeletos, Huo, & Sastry, 2020). These business cycle shocks are utilized to construct the IRFs in the same fashion as in (Angeletos, Huo, & Sastry, 2020) and (Vasudevan, Valente, & Wu, 2022).

**Empirical Facts:** In this section, we present several observations from the SPF data. We begin our analysis by testing the certainty equivalence condition in the survey data. In a standard PIH model, the Euler equation reduces to the certainty equivalence equation such that the expected future consumption level is equal to the current period's consumption. It is indeed this property that makes the consumption growth process a random walk without any predictability. A natural question is whether the certainty equivalence holds in the survey expectation data, which is examined in our first test. The rest of the tests in this section show how the forecast error, at both the consensus level and individual level, reacts to different pieces of news including the income news implied in the forecast revisions and the identified business cycle innovations. We also present the dynamics of the forecast reactions such as the IRFs at different lags. Studying the IRFs is crucial; if consumption growth does not follow a random walk, it may contain past information regarding realized income. The IRFs of the forecast errors can inform us whether there are systematic errors when people form beliefs about the future income process and thus make a sub-optimal consumption-saving plan, which then leads to deviations from the PIH's predictions. In the final part of this section, we summarize all the empirical findings from the survey forecast data and some motivations for an alternative behavioral model.

**Fact 1 Certainty Equivalence Holds in the Survey Forecast:** We run the certainty equivalence regression in the following form,

$$F_t(c_{t+j}) = \alpha + \beta c_t + \varepsilon_{t+j},$$

where the LHS is the consensus forecast of aggregate consumption at different horizons and the RHS predictor is the realized actual consumption level. For the individual-level regression, we replace the predicted variable by $F_{it}(c_{t+j})$ which represents different individuals' forecasts. Specifically, we regress the forecast of 1-, 2-, 3- and 4- 4-quarters ahead conditional on the time-$t$ information set on the consumption level at time $t$. The PIH's Euler equation predicts that $\alpha = 0$ and $\beta = 1$. The regression results are summarized in Table 1 and Table 2. The regression results suggest that the slope is very close to one and is highly significant at conventional levels. In addition, we are not able to reject the hypothesis that the true intercept is different from zero. This holds both

[6] GDP forecasts are used as proxies for forecasters' income expectations.

[7] Namely, only forecasters with at least ten observations are kept. For each quarter-forecast horizon, forecasts that are more than five interquartile ranges away from the median are identified as outliers and are winsorized.

[8] The consensus-level forecast error is the realized value minus the consensus-forecast and the consensus-level forecast revision is the consensus-level forecast change from the last quarter.

for consensus forecast regressions and individual-level forecast regressions. It suggests that incorrect or non-rational beliefs may play an important role in explaining the failure of the PIH and thus the consumption puzzles laid out in the last section. It may be that the survey-based beliefs capture some systematic expectational errors that make consumption growth predictable. Furthermore, the finding provides a useful benchmark as a stylized fact that supports the starting point of our theory, i.e., that the PIH Euler equation holds if we incorporate survey-based beliefs.

**Fact 2 Consumption Growth is Highly Correlated with Income Forecast Error:** If consumption growth is not a random walk, can the survey-based forecast error predict it? Our second fact from the SPF data answers this question. We run the following consumption growth prediction regression,
$\Delta c_{t+j} = \alpha + \beta FE_{t,t+j} + controls + \varepsilon_t$
where $j$ corresponds to a quarter, consumption growth is defined as $\Delta c_{t+j} = c_{t+j} - c_t$ and the consensus forecast error of real income $FE_{t,t+j}$ is defined as $x_{t+j} - \bar{E}_t(x_{t+j})$, for $j$ from 1 to 4. For control variables, we use both past consumption growth and past forecast error at the consensus level. The regression results can be found in Table 3 in the Appendix. We can see that real consumption growth defined at all horizons is highly correlated with the average forecast error of real income. However, this result does not contradict the PIH. In the rational-PIH model, the forecast error of the permanent income component is a sufficient statistic for aggregate consumption growth. Our regression results are consistent with this prediction. However, it is interesting to see that the survey-based forecast or forecast error can generate the predictability of the consumption growth process. This fact should be viewed jointly with Fact 3 presented in the next section. Our second observation mainly shows that consumption growth can be predicted by income forecast errors and the next fact suggests that the forecast error itself can be predicted by the forecast revision or news about future income. Rational consumers fully incorporate any news into their optimal consumption-saving plan. Behavioral consumers may not fully react to the news which therefore makes their consumption predictable by past information. In other words, consumption adjusts with a delay.[9]

**Fact 3 Underreaction and Overreaction in Income and Consumption Forecasts:** Our third piece of motivating evidence tests information rigidity in the SPF forecasts. Specifically, we regress forecast errors on forecast revisions, using both consensus-level observations (as in (Coibion & Gorodnichenko, 2015)) and individual forecaster-level observations (as in (Bordalo, Gennaioli, Ma, & Shleifer, 2020))
In particular, regressions are of the form,
$FE_t = \alpha + \beta_{CG} FR_t + \varepsilon_t$
$FEit = \alpha + \beta BGMSFRit + \varepsilon it$
where we define
$FEt = xt{+}k - \bar{E}t(xt{+}k)$
$FRt = \bar{E}t(xt{+}k) - \bar{E}t{-}k(xt{+}k)$
$FEit = xt{+}k - Eit(xt{+}k)$ $FRit = Eit(xt{+}k) - Eit{-}k(xt{+}k)$.
where $x_{t+k}$ is the variable of interest, such as nominal GDP, $\bar{E}_t(x_{t+k})$ is the consensus forecast for $k$ quarters ahead conditional on the time $t$ information set and $Eit(x_{t+k})$ is forecaster $i$'s forecast in period $t$. We are interested in the sign of $\beta_{CG}$ and $\beta_{BGMS}$. As mentioned in Coibion and Gorodnichenko (2015), $\beta > 0$ corresponds to forecasters under-reacting to information that arrives between period $t-k$ and period $t$, and $\beta < 0$ corresponds to forecasters overreacting to information that arrives between $t-k$ and $t$, with larger magnitude coefficients indicating more under-reaction or overreaction.

A positive coefficient indicates that the forecast error is positively correlated with changes in forecasters' expectations from $t - k$ to $t$. This reflects that forecasters' beliefs did not move sufficiently to capture the information they observed, consistent with under-reaction. Conversely, a negative coefficient indicates that forecasters' beliefs moved too much, consistent with overreaction. The regression results can be found in Table 4 and Table 5. Similar results are also reported in (Bordalo, Gennaioli, Ma, & Shleifer, 2020) and we include them in this paper using an updated data sample. The regression results suggest that individuals consistently overreact to new information when they forecast all three variables. For consensus-level forecasts of real consumption, we do not obtain statistically significant results. However, for both nominal and real GDP, we can see that the CG regression coefficients are positive and statistically significant except for one-quarter ahead

[9] See (Campbell & Deaton, 1989) for a more detailed discussion.

forecasts. We interpret the positive signs as indicating under-reaction to news such that forecasters do not fully adjust their forecasts when receiving new information about the data. These results are consistent with (Bordalo, Gennaioli, Ma, & Shleifer, 2020).

**Fact 4 Initial Under-reaction and Delayed Overreaction in Consensus Income Expectations**: To study how forecasters respond to the arrival of news, we borrow two types of news (Angeletos, Huo, & Sastry, 2020).[10] The shocks are constructed by running a VAR of ten US macroeconomic variables and extracting the linear combination of residuals in the VAR that explains the most quarterly variation of a given macroeconomic variable for 6 to 32 quarters ahead. The first shock, which is called the "main business cycle shock", is constructed by maximizing its contribution to the business cycle variation in unemployment. The authors argue that it is one of the main drivers of the bulk of the business cycle in the data. The second shock we consider is the TFP shock constructed by targeting the TFP series, which is shown to be unrelated to the main business cycle shock at all frequencies. Figure 1 plots the impulse response functions of the consensus forecast error of real and nominal income from the SPF at the quarterly frequency. The impulse response functions are estimated from regressions of the form $yt+h = \alpha h + \beta h\varepsilon t + \gamma hCt + ut+h$

Where $y_{t+h}$ is the variable of interest (forecast error in our case), $C_t$ is the lagged values of forecasts and outcomes used as controls, and $\varepsilon_t$ is the main business cycle shock or the TFP shock. The forecast error is defined as $y_{t+h}$ = $x_{t+h}$-$E_{t+h-j}x_{t+h}$ (the consensus forecast error of the income). We specify $h$ = 1,2,...,20 which corresponds to up to 20 quarters ahead. The sample for the analysis runs from 1968 to 2017.[11] The figure also plots plus and minus one standard error band for the impulse response functions. The impulse response functions reveal that for around five quarters after the arrival of a main business cycle shock and around ten quarters after the arrival of a TFP shock, the consensus forecast of real or nominal income is persistently lower than the realized values, indicating under-reaction to news. However, after that, the forecasted income exceeds the realized income, indicating the subsequent overreaction of the forecasts. These patterns are captured by initial positive forecast errors, followed by negative forecast errors. Similar behaviors are documented for inflation/unemployment forecasts (Angeletos, Huo, & Sastry, 2020) and U.S. interest rate/interest rate differential forecasts (Vasudevan et al. (2022)). ial under-reaction and delayed overreaction of survey-based consensus expectations to news shocks.

**Fact 5 Initial Under-reaction and Delayed Overreaction in Consumption Growth:** From our empirical fact 2, we see that forecast errors of income at the consensus level predict consumption at multiple horizons; we also showed that consensus income forecast errors under-react and then overreact to news. A natural question to ask is whether realized consumption growth itself exhibits a similar pattern of underreaction followed by overreaction. We follow the local projection regression method described in the last section and plot the IRFs of annual consumption growth to the news shocks in Figure 2.[12] We see that consumption growth at an annual frequency also displays the pattern of initial underreaction and subsequent overreaction after the realization of the news. That is, when receiving a positive (negative) main business cycle shock or a TFP shock, consumption growth increases (decreases) in the short term. However, after several quarters, consumption growth becomes negative (positive). We interpret the predictability of aggregate consumption growth, jointly from all the facts we documented, from the belief updating property of the forecasters: they do not fully adjust their forecasts or over-respond in their forecasts, which is reflected in their reported consumption forecasts and realized consumption profiles.

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[10] We downloaded the data on shocks from George-Marios Angeletos' website.

[11] The sample from (Angeletos, Huo, & Sastry, 2020) ends in 2017Q4.

[12] Specifically we estimate the following equation

$$y_{t+h} = \alpha_h + \beta_h \varepsilon_t + \gamma_h C_t + u_{t+h}$$

where $y_{t+h}$ is the annual consumption growth at time $t + h$, $C_t$ are lagged values of forecasts and outcomes used as controls at time $t$, and $\varepsilon_t$ are the main business cycle shock or the TFP shock in the period of $t$.

## 3. A Life-Cycle Consumption Model

We present a theory of consumption in this section. Our goal is to explain consumption behavior in a manner consistent with the motivating empirical evidence and the puzzling observations regarding consumption. The theory we introduce is a standard consumption-saving optimization model with an exogenous income process and incomplete information. Consumers choose their consumption in each period over their lifecycle but are uncertain about the permanent income component. They are endowed with a noisy signal from which they make inferences about the unobserved latent income. In the model, the consumers are also not informed about the structural parameters when updating their beliefs in each period of time. Therefore, they use some subjective parameters when making inferences. We first introduce the model environment and solve the Full Information Rational Benchmark (PIH) in which there is no uncertainty in the market. We then show that the PIH predicts that the consumption change over time cannot be predicted by past information. Finally, we present the full model solution and study how consumption changes react to income news in both the short term and medium term.

**Preliminaries:** Time is discrete and indexed by $t = 0,1,2,...T$. There are an infinite number of ex-ante identical households who are indexed by $i$; we normalize the mass of households to one. In each period $t$, household $i$ earns exogenous income $y_{it}$ and makes consumption and saving plans by maximizing the following lifetime utility,

$$\mathbb{E}_{it}\left(\sum_{j=t}^{T}\beta^{j-t}u(c_{ij})\right)$$

subject to the budget constraint

$$c_{it}+s_{it}=y_{it}+\frac{1}{1+r}s_{it+1}$$

where we denote consumption and saving by $c_{it}$ and $s_{it}$, respectively. The discount factor is denoted by $\beta$ and $r$ is the net return on savings, which is exogenous to all the households. In the rest of the paper, we assume the household's utility function is quadratic and has the following functional form,

$u(c) = -(c - \gamma)^2$,

where $\gamma$ is a fixed and exogenous parameter. Under this assumption, it is easy to show that the optimal consumption profile satisfies the following Euler equation,

$cit = \mathrm{E}it(cit+1)$

where we assume that the discount factor satisfies $\beta(1 + r) = 1$. Combining this equation with the budget constraint, we can write the optimal consumption and consumption growth as

$$c_{it}=-\frac{1-\beta}{1-\beta^{T-t+1}}s_t+\frac{1-\beta}{1-\beta^{T-t+1}}\sum_{j=0}^{T-t}\beta^j\mathbb{E}_{it}(y_{it+j})$$

$$c_{it}-c_{it-1}=\left(\frac{1-\beta}{1-\beta^{T-t+1}}\right)\sum_{j=0}^{T-t}\beta^j\left(\mathbb{E}_{it}(y_{it+j})-\mathbb{E}_{it-1}(y_{it+j})\right)$$

**Income Process and Information:** Throughout the paper, we assume that households earn permanent income with idiosyncratic risk in each period. Specifically, we assume that $y_{it}$ is generated by the following process

$y_{it} = y_t + \sigma_v v_{it}$, $v_{it}$ *i.i.d.* $\sim \mathrm{N}(0,1)$

where $\sigma_v$ is the standard deviation. The permanent income component follows an $AR(1)$ process

$y_t = \rho y_{t-1} + \varepsilon_t$, $\varepsilon_t$ *i.i.d.* $\sim \mathrm{N}(0,1)$

where $\rho \leq 1$ and we normalize the permanent shock to have unit variance. We suppose that households do not directly observe $y_t$ but only observe $y_{it}$. In addition, they do not observe other households' income. We denote the information set for household $i$ at time $t$ by $\mathrm{I}_{it} = \{y_{i0},y_{i1},...,y_{it}\}$.

**Full Information Rational Benchmark:** We first derive the full information benchmark where agents perfectly observe $y_t$. When all households have the same information set, we can effectively suppose there is only a representative agent and aggregate-level consumption coincides with individual-level consumption. It is easy to prove the following proposition.

**Proposition 1:** *If households perfectly observe the process of permanent income, consumption growth satisfies the following equation,*

$$\Delta c_t^* = c_t^* - c_{t-1}^* = \left(\frac{1-\beta}{1-\beta^{T-t+1}}\right)\left(\frac{1-(\rho\beta)^{T-t+1}}{1-\rho\beta}\right)\varepsilon_t.$$

*The covariance between consumption growth and income change is*

$$cov(\Delta c_t^*, \Delta y_t) = \left(\frac{1-\beta}{1-\beta^{T-t+1}}\right)\left(\frac{1-(\rho\beta)^{T-t+1}}{1-\rho\beta}\right).$$

From Proposition 1, it is clear that consumption growth is pure noise and is not predicted by any past information. The intuition is that households want to smooth their consumption flow and consumption mainly depends on expected lifetime income, which is much smoother than income in a particular period. Therefore, consumption growth is mainly the forecast error of lifetime income, which is orthogonal to past variables such as past income or past income change.

**Full Model Solution**: We now present the main result of the paper. In the model where households do not directly observe permanent income, they rely on the income realizations $y_{it}$ to learn so that $y_{it}$ is effectively a noisy signal of $y_t$. We also assume that households' subjective persistence of permanent income may be different from the true parameter. Specifically, we allow households to perceive the autocorrelation of $y_t$ to be $\tilde{\rho}$. We regard $\tilde{\rho}$ as a parameter summarizing extrapolative expectations and will assume $1 \geq \tilde{\rho} \geq \rho$ in the rest of the paper, which we call over-extrapolation of realized income. The following proposition summarizes households' forecast rule of future income and consumption growth.

**Proposition 2:** *If households do not perfectly observe the process of permanent income, their prediction of permanent income satisfies*

$$\tilde{\mathbb{E}}_{it}(y_t) = \left(1-\frac{\lambda}{\tilde{\rho}}\right)\frac{1}{1-\lambda\mathbb{L}}y_{it}$$

*Where*

$$\lambda = \frac{1}{2}\left(\tilde{\rho} + \frac{1+\tau_\nu}{\tilde{\rho}} - \sqrt{(\tilde{\rho} + \frac{1+\tau_\nu}{\tilde{\rho}})^2 - 4}\right) \text{ and } \tau_\nu = \sigma_\nu^{-2}.$$

*Consumption growth satisfies the following equation*

$$\Delta c_{it} = c_{it} - c_{it-1} = \left(\frac{1-\beta}{1-\beta^{T-t+1}}\right)\left(\frac{1-(\tilde{\rho}\beta)^{T-t+1}}{1-\tilde{\rho}\beta}\right)\left(1-\frac{\lambda}{\tilde{\rho}}\right)\frac{1-\tilde{\rho}\mathbb{L}}{1-\lambda\mathbb{L}}y_{it}.$$

The proof can be found in the Appendix. From the above result, we can see that all the information contained in $y_{it}$, not just the current realization of innovation, enters into the computation of consumption growth. To see more clearly how households forecast permanent income, we can rewrite the forecast rule as

$$\tilde{\mathbb{E}}_{it}(y_t) = \frac{\lambda}{\tilde{\rho}}\tilde{\mathbb{E}}_{it-1}(y_t) + \left(1-\frac{\lambda}{\tilde{\rho}}\right)y_{it}.$$

From the above equation, we can see that households' prediction of the fundamental variable is a weighted average of their past forecasts and the new information received. The weight $\left(1-\frac{\lambda}{\tilde{\rho}}\right)$ is the Kalman gain and the forecast itself is an $AR(1)$ process. We can view the parameter $\lambda$ as the informational friction that induces sticky updating of households' beliefs. When households have full information about permanent income, $\lambda$ goes to zero and the current prediction reflects only the new observation $y_{it}$ but no longer depends on the past prediction. In the next section, we formally derive the covariance between forecast error and income news, which will shed light on how households react to income innovations.

**Theoretical Predictions:** In this section, we present a set of predictions from the model developed in the last section. We will mainly focus on the full model with dispersed information and extrapolation since the FIRE benchmark is rather clear and intuitive. We first derive how households' expectation error of future income reacts to income news and show how the model is related to the consumption anomalies we study in this paper.

**The Impulse Response of Forecast Errors:** From the individual household's forecast rule, we define the consensus forecast as the average forecast across households,

$$\bar{\mathbb{E}}_t(y_{t+1}) = (\tilde{\rho} - \lambda)\frac{1}{1-\lambda\mathbb{L}}y_t.$$

We define the consensus- and individual-level news in each period as follows.

**Definition 5.1.** In period *t*, the consensus-level news is the innovation in the permanent income process such that

$news_t = \varepsilon_t$,

and the idiosyncratic-level news is the private noise in $y_{it}$ weighted by its standard deviation

$news_{it} = \sigma_\nu \nu_{it}$.

The following proposition summarizes the Impulse Response Functions (IRFs) of forecast errors with respect to both current news and past news at both the consensus level and individual level. **Proposition 3.** *The one-period ahead forecast errors at the consensus level and individual level are*

$$FE_{t,t+1} = \frac{1-\tilde{\rho}\mathbb{L}}{(1-\rho\mathbb{L})(1-\lambda\mathbb{L})}\varepsilon_{t+1}$$

*and*

$$FE_{it,t+1} = \frac{1-\tilde{\rho}\mathbb{L}}{(1-\rho\mathbb{L})(1-\lambda\mathbb{L})}\varepsilon_{t+1} - \frac{\tilde{\rho}-\lambda}{1-\lambda\mathbb{L}}\sigma_\nu\nu_{it}.$$

*The IRFs of the consensus-level forecast error w.r.t. $news_{t-j}$ are*

$$IRF_t^j = \frac{\partial FE_{t,t+1}}{news_{t-j}} = \frac{\lambda^{j+1}(\tilde{\rho}-\lambda) - \rho^{j+1}(\tilde{\rho}-\rho)}{\rho-\lambda},\ \forall j \geq 0.$$

*The IRFs of the individual-level forecast error w.r.t. $news_{it-j}$ are*

$$\frac{\partial FE_{it,t+1}}{news_{it-j}} = -\lambda^j(\tilde{\rho}-\lambda),\ \forall j \geq 0.$$

*We view the total response of the individual-level forecast errors to news as the aggregate effect of $news_{t-j}$ and $news_{it-j}$ such that*

$$IRF_{it}^j = \frac{\partial FE_{it,t+1}}{news_{t-j}} + \frac{\partial FE_{it,t+1}}{news_{it-j}} = \frac{\lambda^{j+1}(\tilde{\rho}-\lambda) - \rho^{j+1}(\tilde{\rho}-\rho)}{\rho-\lambda} - \lambda^j(\tilde{\rho}-\lambda).$$

The proof can be found in the Appendix. Our first observation is that $IRF^j_{it}$ will always be more negative compared to $IRF^j_t$ given that $\lambda < \tilde{\rho}$, which is easy to prove. This implies that good news at the individual level will result in a more negative forecast error, so households underreact less or overreact more to news at the individual level compared to an average household. Economically, it is due to independent idiosyncratic shocks across periods. When there is some good idiosyncratic news ($\nu_{it}$) realized in the current period, household *i* updates its forecast of future income upwards. However, the good news is only temporary and does not affect the next period's income, which on average disappoints those households and results in a smaller forecast error. Our second observation is that whether households under or overreact to current news depends on the trade-off between extrapolation degree and informational friction. The consensus and individual level forecast error to current news (taking $j = 0$) is $\lambda - (\tilde{\rho} - \rho)$ and $2\lambda - (\tilde{\rho} - \rho) - \tilde{\rho}$ where $\lambda$ arises due to noise in the realized income $y_{it}$. Ceteris paribus, a higher level of extrapolation reduces the reaction of forecast errors to current news and induces more overreaction to news. Noisy information, on the other hand, increases the IRF coefficients and produces more underreaction to news. Which effect dominates depends on the relative strength of noisy learning and extrapolative beliefs. Furthermore, the more auto-correlated the permanent income process is, the more a household forecast underreacts and the less it overreacts. It is also intuitive because a larger value of $\rho$ implies a relatively smaller level of extrapolation ($\tilde{\rho}$ closer to $\rho$) and noisy information plays a more important role.

**Relation to Consumption Anomalies:** Recall that the MPC in the rational-expectation PIH is a function of both the discount factor and the auto-correlation coefficient such that,

$$MPC^* = \left(\frac{1-\beta}{1-\beta^{T-t+1}}\right)\left(\frac{1-(\rho\beta)^{T-t+1}}{1-\rho\beta}\right).$$

One can prove that this MPC is always less than one and increases when households approach the end of the lifecycle. This is because households mainly care about the average lifetime income and when an income shock occurs its average impact will be more pronounced when there are fewer periods left. When the income process is a martingale process, the PIH predicts that the MPC is one. We define consumption as excessively smooth and excessively sensitive relative to the PIH as follows:

**Definition 5.2.** Aggregate consumption is excessively smooth if $MPC = \frac{\partial \Delta c_t}{\partial \varepsilon_t}$ is smaller than $MPC^* = \left(\frac{1-\beta}{1-\beta^{T-t+1}}\right)\left(\frac{1-(\rho\beta)^{T-t+1}}{1-\rho\beta}\right)$ for all $t$. Consumption is excessively sensitive if $\frac{\partial \Delta c_t}{\partial \varepsilon_{t-1}} > 0$ for all $t$.

We can prove that the full model predicts excessively smooth and excessively sensitive consumption growth.

**Proposition 4.** *Aggregate consumption is excessively smooth if the following condition holds*

$$\lambda > \tilde{\rho}\frac{\sum_{j=0}^{T}(\tilde{\rho}^j - \rho^j)\beta^j}{\sum_{j=0}^{T}(\tilde{\rho}\beta)^j}.$$

*When $\tilde{\rho} = \rho$ and $\tau_v < +\infty$, aggregate consumption is always excessively smooth.*
*Aggregate consumption is excessively sensitive if the following condition holds*
*$\tau_v < +\infty$ and $\lambda > \tilde{\rho} - \rho$.*

Note that the condition to generate excess sensitivity corresponds to the condition under which the forecast error under-reacts to a current income shock. The intuition is that if households under-react to current shocks when updating their beliefs concerning average lifetime income, the incorporation of news into their consumption profile is slowed. As a result, good news realized in the current period predicts positive consumption growth in the next period. This causes consumption growth to be correlated with predicted income growth. The PIH fails to predict excessively sensitive consumption growth because income innovations are fully incorporated into consumption in each period and future consumption growth is therefore not predicted by past income news. For consumption to be excessively smooth, however, under-reaction to current income news is not sufficient. To see this, we can decompose the consumption growth as follows,

$$\begin{aligned}\Delta c_t &= \left(\frac{1-\beta}{1-\beta^{T-t+1}}\right)\left(y_t - \bar{\mathbb{E}}_{t-1}(y_t) + \sum_{j=1}^{T-t}\beta^j(\bar{\mathbb{E}}_t(y_{t+j}) - \bar{\mathbb{E}}_{t-1}(y_{t+j}))\right) \\ &= \left(\frac{1-\beta}{1-\beta^{T-t+1}}\right)\left(\underbrace{y_t - \bar{\mathbb{E}}_{t-1}(y_t)}_{\text{Forecast Error}} + \beta\frac{1-(\beta\tilde{\rho})^{T-t}}{1-\beta\tilde{\rho}}\underbrace{(\bar{\mathbb{E}}_t(y_{t+1} - (\bar{\mathbb{E}}_{t-1}(y_{t+1}))}_{\text{Forecast Revision}}\right)\end{aligned}$$

Where the current income news enters both the forecast error and the forecast revision and $\frac{\partial \bar{\mathbb{E}}_t(y_{t+1} - (\bar{\mathbb{E}}_{t-1}(y_{t+1})}{\partial \varepsilon_t} = \tilde{\rho} - \lambda$ compared to $\rho$ in PIH. Under-reacting to news will make the forecast revision a positive function of income innovations $\varepsilon_t$ and a less precise signal (larger $\lambda$) induces a somewhat smaller effect of income news in the second term. However, extrapolation mitigates the sluggishness as summarized in the coefficient that increases with $\tilde{\rho}$. Excess smoothness in aggregate consumption relative to the PIH arises when the average effect of noise is stronger than that of extrapolation. When $\rho = \tilde{\rho}$, our model always predicts a smaller MPC purely due to imperfect observation of the permanent income component. In fact, our model predicts not only excess sensitivity of consumption to recent news but also implies that all past income news affects the consumption growth path. This is because the forecast error in our model is not pure noise and is predictable by past realized news. It is this feature that produces predictable consumption growth when the PIH fails to replicate. Under some values of the parameters, our model also implies negatively autocorrelated consumption growth in the medium term,

**Definition 5.3.** Aggregate consumption growth is negatively autocorrelated in the medium term if $cov(\Delta c_t, \Delta c_{t-j}) < 0$ for some $j > 1$.

We can prove the following proposition regarding the consumption growth covariance between periods $t$ and $t - j$.

**Proposition 5.** *When $\rho > \tilde{\rho}$ and $\tau_v < +\infty$, aggregate consumption growth is negatively autocorrelated in the medium term such that $cov(\Delta c_t, \Delta c_{t-j}) < 0$ for some $j > 1$.*

This implication is mainly due to our agents' over-extrapolating income shocks, which leads to subsequent overreaction. The intuition is as follows. Regardless of how noisy the private income process is, households always positively react to the current shocks (although they may do so in an under-reacting fashion). This means $\varepsilon_{t-j}$ enters the consumption growth from period $t-j-1$ to period $t-j$ positively. It is also not surprising that the impact of $\varepsilon_{t-j}$ has the largest magnitude among all realized shocks. However, when we have $\rho < \tilde{\rho}$, we know that households ultimately overreact to historical income shocks. For example, $\varepsilon_{t-j}$ negatively predicts consumption growth from $t-1$ to $t$, thus resulting in a negative correlation between $\Delta c_t$ and $\Delta c_{t-j}$.

## 4. Conclusion

In this paper, we propose an explanation for several major macro consumption anomalies. These consumption anomalies are deviations from the rational PIH model in which consumption growth is essentially a random walk and is unpredictable. Specifically, the empirical literature documents that aggregate consumption is too smooth in response to unpredictable income changes and too sensitive in response to predicted income changes. Furthermore, in contrast to the prediction from PIH that past income change and consumption growth are uncorrelated in the medium term, the data show that consumption growth is negatively correlated with past income growth. A hump-shaped consumption profile over the lifecycle in the data also contradicts the random walk assumption. Our theoretical explanation of these puzzling facts relies heavily on an unconventional belief formation process of consumers, guided by survey-based evidence. Our focus on survey-based forecasts of aggregate income as a proxy for market participants' beliefs is motivated by the strong relationship between survey-based forecast errors of income and consumption growth. Focusing on income forecasts, we document that consensus forecasts of nominal and real income initially underreact, and subsequently overreact to news shocks.

Moreover, similar patterns of reaction can also be seen in aggregate consumption growth. We propose a model similar to (Angeletos, Huo, & Sastry, 2020) and, (Vasudevan, Valente, & Wu, 2022). Two key frictions play an important role in the model. The first is that consumers do not directly observe the latent fundamental income variable that governs the realized income in each period, so they need to learn the data-generating process to make inferences. Second, we allow consumers to use a subjective persistence parameter, which can be different from the true parameter, in updating their beliefs. Combining these two assumptions, we are able to generate forecast errors of income and equilibrium consumption growth consistent with the evidence. We also carefully characterize the model solution and show its relation to each of the consumption anomalies. There are also some policy implications from this paper. First of all, consumption equilibrium is determined by a wide range of personal expectations of future macroeconomic variables such as inflation rate and interest rate, which in turn affects the equilibrium aggregate production. Therefore, it will be useful to design belief management tools such as forward guidance, considering distorted beliefs, to reach more efficient market outcomes. Second, consumption is ultimately related to investments such as fixed income, real estate and stock market decisions.

This study can be used and extended to develop policies to reduce information frictions in financial markets to further help consumers and investors to make optimal consumption, investment and saving portfolio choices. We conclude with some thoughts on further directions for work suggested by our analysis. First, the agents in our model are ex-post heterogeneous in the sense that their realized personal income (signals) can be different. However, we do not specify the source of this heterogeneity. It could derive from income sources, working location and matching frictions. Due to the heterogeneity in their received signals, consumers disagree about the same fundamental data-generating process. Understanding this heterogeneity, and especially how individual consumption is related to the source of heterogeneity, is an interesting future direction. It may also help to explain the cross-sectional variations in consumption. Second, we do not make quantitative predictions in the paper. An important next step is to use detailed personal consumption expenditure and income data, jointly with survey-based data, to calibrate the model. This will allow us to make more precise predictions such as the timing of underreaction and overreaction, how much excess sensitivity in consumption is due to overreaction and possibly also suggest some welfare implications. Finally, our theoretical framework is a partial equilibrium model. Endogenizing the income process in the model, for example by introducing a firm sector, can be useful. Such a model can be applied to study other macroeconomic implications such as general equilibrium effects and redistribution policy.[13] We leave these directions for future research.

[13] For example, how consumers form income beliefs regulates the slope of the Keynesian cross and how the aggregate demand responds to monetary policy.

**References**

Angeletos, G. M., Huo, Z. & Sastry, K. A. (2020). Imperfect macroeconomic expectations: Evidence and theory. NBER Macroeconomics Annual, 35.

Attanasio, O. P. & Pavoni, N. (2011). Risk sharing in private information models with asset accumulation: Explaining the excess smoothness of consumption. *Econometrica,* 79(4).

Barberis, N., Greenwood, R., Jin, L. & Shleifer, A. (2015). X-capm: An extrapolative capital asset pricing model. *Journal of Financial Economics,* 115(1), 1–24.

Barberis, N., Greenwood, R., Jin, L. & Shleifer, A. (2018). Extrapolation and bubbles. Journal of Financial Economics.

Beeler, J. & Campbell, J. Y. (2012). The long-run risks model and aggregate asset prices: An empirical assessment. *Critical Finance Review,* 1(1).

Berrada, T. (2006). Incomplete information, heterogeneity, and asset pricing. *Journal of Financial Econometrics,* 4(1).

Bondt, D., F., W. & Thaler, R. (1985). Does the stock market overreact? *The Journal of Finance,* 40(3), 793–805.

Bordalo, P., Gennaioli, N., Ma, Y. & Shleifer, A. (2020). *Overreaction in macroeconomic expectations,* 110(9), 2748–82.

Caballero, R. J. (1990). Consumption puzzles and precautionary savings. *Journal of Monetary Economics,* 25(1), 113-136.

Campbell, J. & Deaton, A. (1989). Why is consumption so smooth. *The Review of Economic Studies,* 56(3).

Campbell, J. & Deaton, A. (n.d.). Why is consumption so smooth? *The Review of Economic Studies,* 56(3).

Carroll, C. D., Crawley, E., Slacalek, J., Tokuoka, K. & White, M. N. (n.d.). Sticky expectations and consumption dynamics. *American Economic Journal: Macroeconomics,* 12(3).

Chetty, R. & Szeidl, A. (2016). Consumption commitments and habit formation. *Econometrica,* 84(2), 855 – 890.

Coibion, O. & Gorodnichenko, Y. (2015). Information rigidity and the expectations formation process: A simple framework and new facts. *American Economic Review,* 105(8), 2644–78.

Cutler, D. M., Poterba, J. M. & Summers, L. H. (1991). Speculative dynamics. *the Review of Economic Studies,* 58(3), 529–546.

da Silveira, R. A. & Woodford, M. (2019). Noisy memory and overreaction to the news. AEA Papers and Proceedings, 109.

Deaton, A. (1986). Life-cycle models of consumption: Is the evidence consistent with the theory? Working Paper.

Fernandez-Villaverde, J. & Krueger, D. (2007). Consumption over the life cycle: Facts from consumer expenditure survey data. *The Review of Economics and Statistics,* 89(3).

Flavin, M. (1981). The adjustment of consumption to changing expectations about future income. *Journal of Political Economy,* 89(5).

Gourinchas, P. O. & Parker, J. A. (2003). Consumption over the life cycle. Econometrica.

Gourinchas, P. O. & Tornell, A. (2004). Exchange rate puzzles and distorted beliefs. *Journal of International Economics,* 64(2), 303–333.

Huang, K. X. & Caliendo, F. (2011). Rationalizing multiple consumption-saving puzzles in a unified framework. *Front. Econ. China,* 6(3), 359–388.

Jappelli, T. & Pistaferri, L. (2010). The consumption response to income changes. Annual Review of Economics, 2.

Kucinskas, S. & Peters, F. S. (2022). Measuring under- and overreaction in expectation formation. The Review of Economics and Statistics.

Liao, J., Peng, C. & Zhu, N. (2021). Extrapolative bubbles and trading volume. The Review of Financial Studies.

Matejka, F. (2016). Rationally inattentive seller: Sales and discrete pricing. *The Review of Economic Studies,* 83(3), 1125–1155.

Nimark, K. (2008). Dynamic pricing and imperfect common knowledge. *Journal of Monetary Economics,* 55(2), 365–382.

Pagel, M. (2017). Expectations-based reference-dependent life-cycle consumption. *Review of Economic Studies,* 84, 885–934.

Reis, R. (2006). Inattentive consumers. *Journal of Monetary Economics,* 53(8), 1761–1800.

Sims, C. A. (2003). Implications of rational inattention. *Journal of Monetary Economics,* 50(3), 665– 690.

Sims, C. A. (2010). Rational inattention and monetary economics, 2, 155–181.

Sims, C. A. (2010). Rational inattention and monetary economics. *Handbook of Monetary Economics,* 3, 155–181.

The adjustment of consumption to changing expectations about future income. (n.d.).
Timmermann, A. (2001). Structural breaks, incomplete information, and stock prices. *Journal of Business and Economic Statistics,* 19(3), 299–314.
Vasudevan, K., Valente, J. P. & Wu, T. (2022). The role of beliefs in asset prices: Evidence from exchange rates. Working Paper.
Zinn, J. (2013). Self-attribution bias and consumption. Working Paper.

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**Appendices**

**A. Tables and Figures**

**Table 1: Regression of Consumption Level Forecast on Realized Consumption (Individual Level)**

| | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| | $E_{it}(c_{t+1})$ | $E_{it}(c_{t+2})$ | $E_{it}(c_{t+3})$ | $E_{it}(c_{t+4})$ |
| $c_t$ | 1.004*** | 1.013*** | 1.020*** | 1.027*** |
| | (0.00450) | (0.00387) | (0.00370) | (0.00369) |
| Constant | 8.437 | -3.311 | -9.404 | -11.03 |
| | (21.92) | (18.81) | (17.96) | (17.92) |
| $N$ | 5475 | 5476 | 5477 | 5452 |

Standard errors in parentheses
$^{*}$ $p < 0.1$, $^{**}$ $p < 0.05$, $^{***}$ $p < 0.01$
This table summarizes the results from regressing individual-level consumption forecast on realized consumption level in the following form
$Fit(c_{t+j}) = \alpha + \beta c_t + \varepsilon_{t+1}$.
The standard errors are clustered at the year-quarter level.

**Table 2: Regression of Consumption Level Forecast on Realized Consumption (Consensus Level)**

| | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| | $E_t(c_{t+1})$ | $E_t(c_{t+2})$ | $E_t(c_{t+3})$ | $E_t(c_{t+4})$ |
| $c_t$ | 1.005*** | 1.013*** | 1.021*** | 1.028*** |
| | (0.00108) | (0.00163) | (0.00225) | (0.00262) |
| Constant | 4.111 | -5.919 | -11.59 | -13.55 |
| | (8.027) | (12.35) | (17.08) | (19.87) |
| $N$ | 163 | 163 | 163 | 163 |

Standard errors in parentheses
$^{*}$ $p < 0.1$, $^{**}$ $p < 0.05$, $^{***}$ $p < 0.01$
This table summarizes the results from regressing consensus-level consumption forecast on realized consumption level in the following form
$\bar{F}t(ct+j) = \alpha + \beta ct + \varepsilon t+1$.
We report the HAC-standard errors in the table.

**Table 3: Regression of Consumption Growth on Real Income Forecast Error**

| | Real Income Forecast Errors | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| | 1Q | 2Q | 3Q | 4Q | 1Q | 2Q | 3Q | 4Q |
| Coefficient | 0.711*** | 0.672*** | 0.665*** | 0.692*** | 0.731*** | 0.604*** | 0.543*** | 0.477*** |
| | (0.0873) | (0.0821) | (0.0892) | (0.0931) | (0.0854) | (0.0687) | (0.0577) | (0.0490) |
| Controls | N | N | N | N | Y | Y | Y | Y |
| $N$ | 194 | 194 | 195 | 196 | 192 | 193 | 194 | 195 |

Standard errors in parentheses
$^{*} p < 0.1$, $^{**} p < 0.05$, $^{***} p < 0.01$
This table reports the regression coefficients from the following equation
$\Delta c_{t+j} = \alpha + \beta FE_{t,t+j} + controls + \varepsilon_t$
where $j$ corresponds to a quarter, the consumption growth is defined as $\Delta c_{t+j} = c_{t+j} - c_t$ and consensus forecast error of the real income $FE_{t,t+j}$ is defined as $x_{t+j} - \bar{\mathrm{E}}_t(x_{t+j})$, for $j$ from 1 to 4. For control variables, we use both the past consumption growth and the past forecast error at the consensus level. We report the HAC standard errors in the above table.

**Table 4: Regression of Forecast Errors on Forecast Revisions (Individual Level)**

| | Real Income | | | |
|---|---|---|---|---|
| | (1) | (2) | (3) | (4) |
| | 1Q | 2Q | 3Q | 4Q |
| Coefficient | -0.332*** | -0.165*** | -0.172*** | -0.149*** |
| | (0.0283) | (0.0384) | (0.0468) | (0.0379) |
| $N$ | 5648 | 5657 | 5625 | 4815 |
| $R^2$ | 0.083 | 0.014 | 0.012 | 0.007 |

| | Nominal Income | | | |
|---|---|---|---|---|
| | (1) | (2) | (3) | (4) |
| | 1Q | 2Q | 3Q | 4Q |
| Coefficient | -0.369*** | -0.217*** | -0.181*** | -0.217*** |
| | (0.0247) | (0.0305) | (0.0326) | (0.0345) |
| $N$ | 5719 | 5721 | 5688 | 4938 |
| $R^2$ | 0.112 | 0.027 | 0.015 | 0.019 |

| | Real Consumption | | | |
|---|---|---|---|---|
| | (1) | (2) | (3) | (4) |
| | 1Q | 2Q | 3Q | 4Q |
| Coefficient | -0.361*** | -0.354*** | -0.241*** | -0.243*** |
| | (0.0318) | (0.0365) | (0.0380) | (0.0405) |
| $N$ | 3878 | 3884 | 3867 | 3396 |
| $R^2$ | 0.072 | 0.059 | 0.025 | 0.019 |

Standard errors in parentheses
$^{*} p < 0.1$, $^{**} p < 0.05$, $^{***} p < 0.01$
This table summarizes the results from regressing individual-level forecast error on forecast revision in the following form
$FEit = \alpha + \beta BGMSFRit + \varepsilon t$
The standard errors are double clustered at year-quarter and forecaster ID levels. Columns (1) - (4) represent 1-, 2-, 3and 4-quarter ahead of forecast.

**Table 5: Regression of Forecast Errors on Forecast Revisions (Consensus Level)**

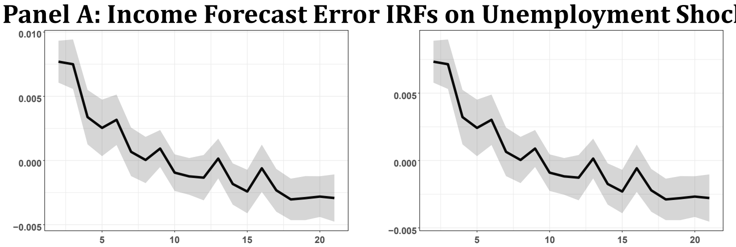
| | Real Income | | | |
|---|---|---|---|---|
| | (1) | (2) | (3) | (4) |
| | 1Q | 2Q | 3Q | 4Q |
| Coefficient | 0.00767 | 0.349** | 0.482*** | 0.450* |
| | (0.0986) | (0.140) | (0.181) | (0.231) |
| $N$ | 196 | 196 | 196 | 191 |
| $R^2$ | 0.000 | 0.034 | 0.048 | 0.027 |

| | Nominal Income | | | |
|---|---|---|---|---|
| | (1) | (2) | (3) | (4) |
| | 1Q | 2Q | 3Q | 4Q |

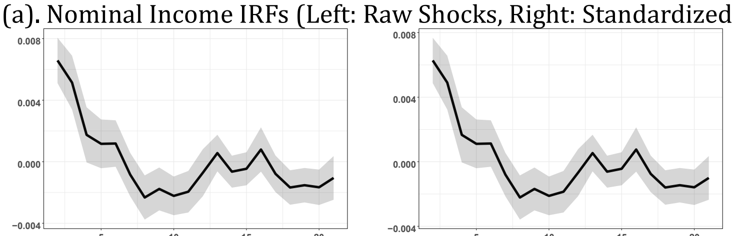
| | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| Coefficient | -0.0448 | 0.319** | 0.512*** | 0.559*** |
| | (0.0901) | (0.126) | (0.164) | (0.208) |
| $N$ | 199 | 199 | 199 | 194 |
| $R^2$ | 0.001 | 0.034 | 0.060 | 0.054 |
| | Real Consumption | | | |
| | (1) | (2) | (3) | (4) |
| | 1Q | 2Q | 3Q | 4Q |
| Coefficient | -0.0475 | -0.0413 | 0.156 | 0.237 |
| | (0.149) | (0.153) | (0.182) | (0.243) |
| $N$ | 145 | 145 | 145 | 145 |
| $R^2$ | 0.001 | 0.001 | 0.006 | 0.009 |

Standard errors in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$
This table summarizes the results from regressing consensus-level forecast error on forecast revision in the following form
$FE_t = \alpha + \beta_{CG}FR_t + \varepsilon_t$
We report the HAC standard errors. Columns (1) - (4) represent 1-, 2-, 3- and 4-quarter ahead forecast.

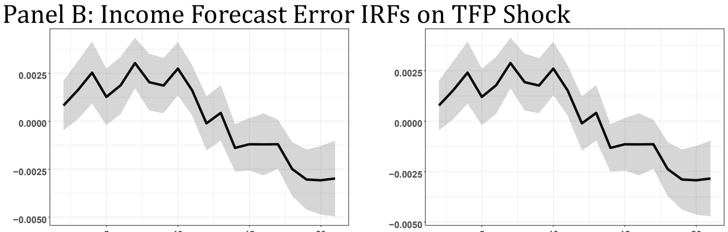
**Figure 1: Under-reaction and Overreaction in Income Expectation to News Shocks**
**Panel A: Income Forecast Error IRFs on Unemployment Shock**



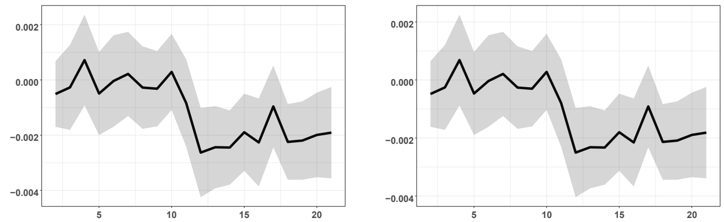

(a). Nominal Income IRFs (Left: Raw Shocks, Right: Standardized Shocks)




(b). Real Income IRFs (Left: Raw Shocks, Right: Standardized Shocks)
Panel B: Income Forecast Error IRFs on TFP Shock



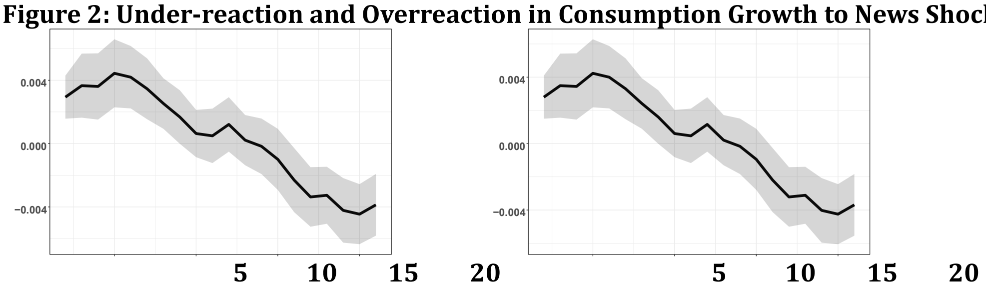

(c). Nominal Income IRFs (Left: Raw Shocks, Right: Standardized Shocks)



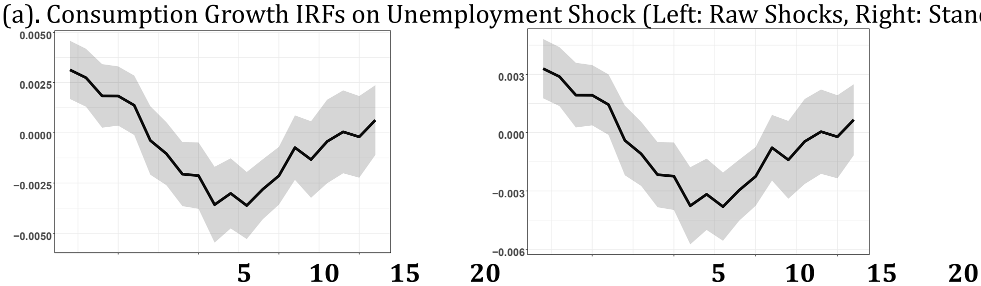
(d). Real Income IRFs (Left: Raw Shocks, Right: Standardized Shocks)

The figure plots impulse response functions (IRFs) of US income consensus forecast errors in response to news shocks. The IRFs are estimated from regressions of the form $y_{t+h} = \alpha_h + \beta_h\varepsilon_t + \gamma_h C_t + u_{t+h}$ where $y_{t+h}$ is forecast error, $C_t$ are lagged values of forecasts and outcomes used as controls, and $\varepsilon_t$ are the main business cycle shock or the TFP shock. The forecast error is defined as $y_{t+h} = x_{t+h} - \bar{E}_{t+h-j} x_{t+h}$ (the consensus forecast error of the income). The sample for the analysis runs from 1968 to 2017. The left column in the figure plots the raw shock and the right column plots the standardized shocks which transform the raw shocks by subtracting the mean and dividing by the standard deviations over the sample.

**Figure 2: Under-reaction and Overreaction in Consumption Growth to News Shocks**




(a). Consumption Growth IRFs on Unemployment Shock (Left: Raw Shocks, Right: Standardized Shocks)




(b). Consumption Growth IRFs on TFP Shock (Left: Raw Shocks, Right: Standardized Shocks)

The figure plots impulse response functions (IRFs) of consumption growth in response to news shocks. The IRFs are estimated from regressions of the form $y_{t+h} = \alpha_h + \beta_h\varepsilon_t + \gamma_h C_t + u_{t+h}$ where $x_{t+h}$ is the annual consumption change, $C_t$ are lagged values of forecasts and outcomes used as controls, and $\varepsilon_t$ are the main business cycle shock or the TFP shock. The sample for the analysis runs from 1968 to 2017. The left column in the figure plots the raw shock and the right column plots the standardized shocks which transform the raw shocks by subtracting the mean and dividing by the standard deviations over the sample.

**B. Proofs**

**Proof of Proposition 2**

The income process can be written as

$$s_{it} = M(\mathbb{L}) \begin{bmatrix} \varepsilon_t \\ \tilde{\nu}_{it} \end{bmatrix}$$

where $M(\mathbb{L}) = \begin{bmatrix} \frac{1}{1-\tilde{\rho}\mathbb{L}} & \tau_\nu^{-1/2} \end{bmatrix}$ and $\tilde{\nu}_{it}$ has a variance of one. The fundamental representation is

$$B(\mathbb{L}) = \tau_\nu^{-1/2} \sqrt{\frac{\tilde{\rho}}{\lambda}} \frac{1-\lambda\mathbb{L}}{1-\tilde{\rho}\mathbb{L}}$$

where

$$\lambda = \frac{1}{2}\Big(\tilde{\rho} + \frac{1+\tau_\nu}{\tilde{\rho}} - \sqrt{(\tilde{\rho} + \frac{1+\tau_\nu}{\tilde{\rho}})^2 - 4}\Big).$$

By the Wiener-Hopf prediction formula, the individual forecast formula is

$$\tilde{\mathbb{E}}_{it}(y_t) = \left[\frac{1}{1-\tilde{\rho}\mathbb{L}} M^T(\mathbb{L}^{-1}) B(\mathbb{L}^{-1})^{-1}\right]_+ B(\mathbb{L})^{-1} y_{it} = \left(1 - \frac{\lambda}{\tilde{\rho}}\right) \frac{1}{1-\lambda\mathbb{L}} y_{it}.$$

And

$\tilde{E}_{it}(y_{t+j}) = \tilde{E}_{it}(y_{it+j}) = \tilde{\rho}^j \tilde{E}_{it}(y_t)$

when $j \geq 1$.

**Proof of Proposition 3**

The forecast error at the consensus level can be written as

$$FE_{t,t+1} = y_{t+1} - \bar{\mathbb{E}}_t(y_{t+1}) = \frac{1}{1-\rho\mathbb{L}}\varepsilon_{t+1} - \frac{(\tilde{\rho}-\lambda)}{1-\lambda\mathbb{L}} \frac{1}{1-\rho\mathbb{L}}\varepsilon_t = \frac{1-\tilde{\rho}\mathbb{L}}{(1-\rho\mathbb{L})(1-\lambda\mathbb{L})}\varepsilon_{t+1},$$

Similarly, the forecast error at the individual level is

$$FE_{it,t+1} = y_{t+1} - \tilde{\mathbb{E}}_{it}(y_{t+1}) = \frac{1-\tilde{\rho}\mathbb{L}}{(1-\rho\mathbb{L})(1-\lambda\mathbb{L})}\varepsilon_{t+1} - \frac{\tilde{\rho}-\lambda}{1-\lambda\mathbb{L}}\sigma_\nu \nu_{it}.$$

The response of consensus level forecast error to news is

$$\frac{\partial FE_{t,t+1}}{news_{t-j}} = (\rho^{j+1} + \rho^j\lambda + \ldots + \lambda^{j+1}) - \tilde{\rho}(\rho^j + \rho^{j-1}\lambda + \ldots + \lambda^j) = \frac{\lambda^{j+1}(\tilde{\rho}-\lambda) - \rho^{j+1}(\tilde{\rho}-\rho)}{\rho-\lambda}$$

for all $j \geq 0$. The response of individual-level forecast errors to private news is

$$\frac{\partial FE_{it,t+1}}{news_{it-j}} = -\lambda^j(\tilde{\rho}-\lambda)$$

for all $j \geq 0$.

**Proof of Proposition 4**

To have excessively sensitive aggregate consumption, we need to show that

$$\frac{\partial \Delta c_t}{\partial \varepsilon_{t-1}} = \left(\frac{1-\beta}{1-\beta^{T-t+1}}\right)\left(\frac{1-(\tilde{\rho}\beta)^{T-t+1}}{1-\tilde{\rho}\beta}\right)\left(1-\frac{\lambda}{\tilde{\rho}}\right)(\rho+\lambda-\tilde{\rho}) \propto (\rho+\lambda-\tilde{\rho})$$

which is positive if $\lambda > \tilde{\rho} - \rho$. To have excessively smooth, we need to show

$$MPC = \left(\frac{1-\beta}{1-\beta^{T-t+1}}\right)\left(\frac{1-(\tilde{\rho}\beta)^{T-t+1}}{1-\tilde{\rho}\beta}\right)\left(1-\frac{\lambda}{\tilde{\rho}}\right)$$

or

$$(1-\frac{\lambda}{\tilde{\rho}}) > \frac{\left(\frac{1-\beta}{1-\beta^{T-t+1}}\right)\left(\frac{1-(\rho\beta)^{T-t+1}}{1-\rho\beta}\right)}{\left(\frac{1-\beta}{1-\beta^{T-t+1}}\right)\left(\frac{1-(\tilde{\rho}\beta)^{T-t+1}}{1-\tilde{\rho}\beta}\right)}$$

which is equivalent to

$$\lambda > \tilde{\rho}\frac{\sum_{j=0}^T(\tilde{\rho}^j - \rho^j)\beta^j}{\sum_{j=0}^T(\tilde{\rho}\beta)^j}.$$

**Proof of Proposition 5**

We can derive the following covariance term between $\Delta c_t$ and $\Delta c_{t-j}$ for an arbitrary $j$

$$cov(\Delta c_t, \Delta c_{t-j}) \propto \sum_{k=j}^{+\infty} \frac{(\tilde{\rho}-\rho)\rho^k + (\lambda-\tilde{\rho})\lambda^k}{\lambda-\rho} \frac{(\tilde{\rho}-\rho)\rho^{k-j} + (\lambda-\tilde{\rho})\lambda^{k-j}}{\lambda-\rho}$$

$$\propto (\tilde{\rho}-\rho)\frac{(\lambda-\rho)(1-\tilde{\rho}\rho)}{1-\rho^2}\rho^j + (\lambda-\tilde{\rho})\frac{(\lambda-\rho)(1-\tilde{\rho}\lambda)}{1-\lambda^2}\lambda^j.$$

This means that

$$(\lambda-\rho)cov(\Delta c_t, \Delta c_{t-j}) \propto (\tilde{\rho}-\rho)\frac{(1-\tilde{\rho}\rho)}{1-\rho^2}\rho^j + (\lambda-\tilde{\rho})\frac{(1-\tilde{\rho}\lambda)}{1-\lambda^2}\lambda^j.$$

Suppose $\tilde{\rho} > \rho$ and $\lambda > 0$ (or $\tau_v < +\infty$) and we can consider two different cases. Suppose $\lambda \geq \rho$, then $(\rho/\lambda)^j$ goes to zero when $j$ goes to infinite. So the RHS must be negative for some $j$ which implies $cov(\Delta c_t, \Delta c_{t-j}) < 0$ for some $j$. The same argument can be made when $\lambda < \rho$. This completes our proof.