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Investigating Chaos on the Johannesburg Stock Exchange

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Abstract: This study investigates the existence of chaos on the Johannesburg Stock Exchange (JSE) and studies three indices namely the FTSE/JSE All Share, FTSE/JSE Top 40 and FTSE/JSE Small Cap. Building upon the Fractal Market Hypothesis to provide evidence on the behavior of returns time series of the above mentioned indices, the BDS test is applied to test for non-random chaotic dynamics and further applies the rescaled range analysis to ascertain randomness, persistence or mean reversion on the JSE. The BDS test shows that all the indices examined in this study do not exhibit randomness. The FTSE/JSE All Share Index and the FTSE/JSE Top 40 exhibit slight reversion to the mean whereas the FTSE/JSE Small Cap exhibits significant persistence and appears to be less risky relative to the FTSE/JSE All Share and FTSE/JSE Top 40contrary to the assertion that small cap indices are riskier than large cap indices.

Keywords: *Fractal Market Hypothesis; Efficient Market Hypothesis; Chaos Theory; Rescaled Range Analysis*

1. Introduction

Financial crises, such as the ones that occurred in 1987, 1998, 2000 and then recently in 2007, have been brushed off as anomalies by proponents of the Efficient Market Hypothesis (EMH) who maintain that markets remain informationally efficient. However, the frequency with which these crises occur cannot be explained by the underlying assumptions of an efficient market. Although a study by Bendel, Smit and Hamman (1996) provides a special impetus on the behaviour of the stock market time series using a variety of indices, results were somehow mixed across indices. However, evidence of long-run persistence in the overall share returns were observed suggesting that future returns are influenced by past returns at least in the long term (Bendel, Smith & Hamman, 1996) which cultivates the need for further interrogation of the behaviour of share returns in modern economies. Classical finance theory is based, inter alia, on the assumptions of investors being rational, of informationally efficient markets and market equilibrium. Equilibrium infers the nonexistence of emotional forces like greed and fear, which trigger the economy to evolve and to adjust to new conditions. Regulating such human tendencies are desirable to minimise their effects, but doing away with them, however, "would take away the life out of the system, including the far from equilibrium conditions that are necessary for development" (Peters, 1996: 5).

This study applies the BDS test as described by Brock, Dechert and Scheinkman (1996) to test for the null hypothesis that the return series of the selected indices are pure noise or completely random. The BDS test, inter alia, has the ability to identify different kinds of deviations from randomness be it non-linear or linear stochastic processes and deterministic chaos. The BDS test is the most popular test for non-linearity and was originally created to test for the null hypothesis of independent and identical distribution (iid) aimed at identifying non-random chaotic dynamics (Zivot & Wang, 2006). The study further applies the rescaled range analysis developed by Hurst (1951) to detect persistence, mean reversion or randomness on the Johannesburg Stock Exchange (JSE) with the aim of providing more adequate assumptions and consequently more realistic models of financial behaviour on the JSE. Closely related to the rescaled range analysis is the Hurst exponent, which is indicated by H, sometimes referred to as 'the index of dependence', which measures three kinds of trends in a given time series, namely, mean reversion, persistence and randomness. The rescaled range analysis was widely used in financial analysis when the application of chaos theory in financial analysis was popular in the early 1990s (Voss, 2013).

As risk remains a fundamental consideration in any investment strategy, an appropriate evaluation of risk based on empirical evidence rather than theoretical postulations will provide practitioners a more comprehensive understanding of risk. Moreover, with the use of fractal statistics, it would be possible to improve financial risk models and provide an alternative discussion of financial markets which differs from the neoclassical assumptions of equilibrium, rationality, perfect markets and the mathematical hypotheses of continuity and symmetry. Chaos Theory and fractal science offers a description of the messiness and the fractal characteristic of financial markets and provide sufficient perspective as well as the mathematical tools required to analyse it. These tools will be beneficial to finance theories as they offer more suitable and realistic assumptions and models of financial market behaviours. This study is conducted on the time series of selected indices on the JSE in South Africa (FTSE/JSE All Share, Top 40 and Small Cap). The JSE is the 19th largest stock exchange in the world by market capitalization, it is the largest and the first stock exchange in Africa established in 1887 during the first gold rush in South African, with 383 listed companies and $ 997.17 billion in market capitalization as at June 2016 (JSE, 2013, World Federation of Exchanges, 2016).South Africa is ranked number one in terms of securities exchange regulations out of 144 countries according to the World Economic Forum's 2014-2015 Global Competitive Index Survey making it the fifth consecutive year the JSE has remained number one in the survey, also ranked number three in the ability to raise capital through the local equity market, number three again in terms of the effectiveness of corporate boards and number two in protecting the rights of minority shareholders (African Securities Exchanges Association, 2016).

2. Literature Review

As financial crises are becoming pervasive, the assumption of efficient markets is increasingly being criticised. Velasquéz (2009) proposes adapting Chaos Theory and Fractal Science to explain financial phenomena. Chaos theory is the study of systems that appear to follow a random behaviour, even though they are actually part of a deterministic process, and the random behaviour is given by their typical sensitivity to initial conditions that leads the system to unpredictable dynamics. One of the founders of chaos theory, Edward Lorenz, summarises this theory elegantly: "Chaos: when the present determines the future, but the approximate present does not approximately determine the future" (Hand 2014: 45). Financial markets are non-linear dynamic systems characterised by positive feedback and fractals, and therefore "what happened yesterday influences what happens today" (Peters, 1996:9). Peters (1996) therefore proposed the Fractal Market Hypothesis (FMH) for modelling financial markets. Benoit Mandelbrot, who is regarded as the father of fractal geometry, first discovered the distinguishing characteristics of fractals in financial time series, but many economists rejected his ideas so he began to lose interest in fractals in finance, and turned to physics. In the field of physics, he developed the fractal geometry of nature (Velasquéz, 2009). Mandelbrot spotted that the variance of prices misbehaved, culminating in abnormally big changes. This behaviour was manifested in "fat-tailed" and high-peak distributions, which commonly followed a power law with the implication that graphs, will not descend toward zero as strikingly as a Gaussian curve. However, the most distinctive property was that these leptokurtic (fat-tailed and high-peak) distributions seemed unchanged irrespective of time scale (weekly, monthly or yearly). Mandelbrot therefore concluded that "the very heart of finance is a fractal" (Mandelbrot & Hudson, 2005:147).

With the underlying classical assumptions of financial markets behaviour being heavily criticised, Buchanan (2013) suggests adopting a disequilibrium view of financial markets, claiming that the disequilibrium view submits that the crashes of 6 May 2010 or of October 1987 or of 2007–2008 were not any more abnormal than the March 2011 earthquake in Japan or the April 1906 quake in San Francisco. Market economies are self-referential and self-propelling systems intensely propelled by expectations and perceptions, and these systems regularly foster explosive amplifying feedbacks. Buchanan (2013) asserts that it is not easy to foretell the instant when a bubble will collapse, and equilibrium economics has resolved, therefore, that bubbles do not exist. A classic example is the refusal of Eugene Fama to admit to the existence of bubbles, for example in an interview in November 2013 on National Public Radio's *Planet Money*. Fama states that the word 'bubble' drives him crazy given that there is nothing to prove that anyone can predict when prices will go down, claiming that markets work and so bubbles cannot be predicted (NPR, 2013). The first comprehensive research on daily stock returns was done by Fama (1965) who discovered that stock returns were negatively skewed; therefore more observations were in the left-hand tail than in the right-hand. Furthermore, the tails appeared fatter and the peak round the mean was higher than what the normal distribution predicted. According to Corhay & Rad (1994), empirical findings reveal the existence of non-linear dependencies that the random walk model fails to explain. Sterge (1989), in an additional study of financial futures prices of treasury bonds, treasury notes and Eurodollar contracts, finds the same leptokurtic distributions. Sterge (1989) notes that "very large (three or more standard deviations from the norm) price changes can be expected to occur two to three times as often as predicted by normality."

McLean & Pontiff (2016) studied the return predictability of 97 factors that academic studies have shown to predict the cross-section of stock returns using out-of-sample and post-publication and found that factors lose 26% of their power after discovery. This inter alia, may be attributed to the effects of data mining. Factors further lose 32% of their predictability power after they appear in academic papers suggesting that investors only learn about this mispricing only after they have been published in academic papers. British hydrologist H.E. Hurst published a paper in 1951 with the title "Long-Term Storage Capacity of Reservoirs", which dealt with modelling of reservoirs, while he was trying to find a way to model the river Nile levels so that architects could build a reservoir of appropriate size (Peters, 1996). This work by Hurst paved the way for a statistical methodology that distinguishes random from non-random systems and for identifying the persistence of trends – a methodology referred to as rescaled range analysis (R/S analysis) (Mansukhani, 2012). While researching the fractal nature of financial markets, Mandelbrot chanced on Hurst's work and recognised it's potential and therefore introduced it to fractal geometry (Mansukhani, 2012). The Hurst exponent measures long-term memory of time series. The exponent relates to the autocorrelations of a given time series, and the rate at which such autocorrelations diminish as the lag between pairs of values increases. According to Peters (1996), a higher value of H depicts less noise and more persistence and a more distinct trend than lower values with higher values showing less risk albeit exhibiting abrupt changes.

On the JSE, Jefferis and Smith (2005), adopting a GARCH methodology with parameters that vary with time, and employing a test of evolving efficiency (TEE) over the period 1990 to 2001, concluded that the JSE is weak form efficient. Adelegan (2003, 2009) finds the JSE to be informationally inefficient, by testing the reaction of market participants to changes in dividend policies of listed firms. Smith (2008), however, rejects the random walk hypothesis on the JSE, using tests of four joint variance ratios. In the following section, we describe the data and the methodology for conducting the BDS test and deriving the Hurst exponent. Section 3 provides the results and discussion of our findings Section 4 concludes the paper and section 5 provides the list of figures referred to in section 3.

3. Methodology

This section discusses the data selected for the study and the methodology the study adopts in testing for non-linearity and chaos on the JSE.

Data: The data for this study were obtained from the database of McGregor BFA, based in Johannesburg, South Africa. McGregor is a prominent provider of stock exchange and accounting data to firms and researchers. McGregor has standardised financial data dating from 1972 to date, and has information for all companies and industries on the JSE. This study investigates the fractal nature of the JSE over the period 15 June 1995 to 12 November 2014. The indices investigated are the daily returns of the FTSE/JSE All Share (J203), which represents 99% of the full market capitalisation of all eligible shares listed on the main board of the JSE; FTSE/JSE Top 40 (J200), which represents the largest 40 companies on the JSE ranked by market capitalisation; and FTSE/JSE Small Cap (J202), which consists of all the remaining companies after the selection of the top 40 and mid cap companies. The study takes 8 cycles of sub-samples from a large sample of n = 4840, with n = 2420 in the second cycle with 2 sub-samples, and so on until 20 sub-samples of n = 242.

The BDS Test: The test for correlation integral is the main concept behind the BDS test (Zivot& Wang, 2006). The correlation integral measures how frequent temporal patterns are repeated in a given time series. The BDS test is designed to spot non-linear dependence (Oppong et al., 1999).

For a given time series x_t for t =1,2,…,T with its m-history as $x_t^m = (x_t, x_{t-1}, \dots, x_{t-m+1})$, we can estimate the correlation integral at embedded dimension m by:

$$C_{m,\epsilon} = \frac{2}{T_m(T_m - 1)} \sum_{m \leq s} \sum_{< t \leq T} I(x_t^m, x_s^m; \epsilon)$$

Where $T_m = T - m + 1$and $I(x_t^m, x_s^m; \epsilon)$represents a signalling function equal to 1 if $|x_{t-1} - x_{s-1}| < \epsilon$ for i = 0,1,…,m – 1 and zero otherwise. Instinctively, the correlation integral is an estimation of the probability that any m-dimensional points being in a distance of ϵ of each other. This implies that it calculates the joint probability:

$$\mathrm{PR}(|x_t - x_s| < \epsilon, |x_{t-1} - x_{s-1}| < \epsilon, \dots, |x_{t-m+1} < x_{s-m+1|} < \epsilon)$$

If x_t are iid, then this probability must be equal to:

$$C_{1,\epsilon}^m = PR(|x_t - x_s| < \epsilon)^m$$

(Brock et al., 1996) define the BDS test as:

$$V_{m,\epsilon} = \sqrt{T} \frac{C_{m,\epsilon} - C_{1,\epsilon}^m}{s_{m,\epsilon}}$$

Where $s_{m,\epsilon}$is the standard deviation of $\sqrt{T}(C_{m,\epsilon} - C_{1,\epsilon}^m)$ and can be consistently estimated, as documented by Brock et al. (Brock et al., 1996). Under conditions of fairly moderate regularity, the BDS test converges in distribution to N(0,1):

$$V_{m,\epsilon} \xrightarrow{d} N(0,1)$$

One advantage of the BDS test is that it requires no distributional assumptions on the series to be tested.

The Hurst Exponent: In proposing the FMH, Peters (1994) applied a modified rescaled range (R/S) procedure, which was pioneered by Hurst (1951). Peters (1994) and Howe, Martin & Wood (1997) review the steps for computing the R/S analysis. First, the index series of the JSE is converted into logarithmic returns, S_t, at time period t of the series of the JSE index. Using raw daily price data in stock markets has many limitations because prices are generally non-stationary (Mehta, 1995) and therefore interfere with estimating the H exponent. The series is therefore converted into logarithmic rates of returns to overcome the problem. In line with Peters (1994), the study divides the time period into A sub-periods with a length of n, so that $A \times n = N$, with N being the length of the series N_t. The study labels each sub-period $I_a$where a = 1,2,3…, A. The study further labels each element in $I_a$is categorised $N_{k,a}$where k= 1,2,3,…,n. The average value, $e_a$for each I_a of length n is defined as:

$$e_a = \left(\frac{1}{n}\right) \times \sum_{k=1}^{n} N_{k,a}$$

The range R_{I_a} is given as the maximum minus the minimum value$X_{k,a}$, within every sub-period I_a given as:

$$R_{I_a} = \max(X_{k,a}) - \min(X_{k,a}), \text{where } 1 \leq k \leq n, 1 \leq a \leq A,$$

with

$$X_{k,a} = \sum_{i=1}^{k} (N_{i,a} - e_a), k = 1,2,3, \dots n,$$

being the time series of the accumulated divergence from the mean for each sub-period. Each range $R_{I_a}$is divided by the sample standard deviation S_{I_a} that corresponds to it to normalise the range. The standard deviation is given as:

$$S_{I_a} = \left[\left(\frac{1}{n}\right) \times \sum_{k=1}^{n} (N_{k,a} - e_a)^2\right]^{0.5}$$

The mean R/S values for length n is given as:

$$(R/S)_n = \left(\frac{1}{A}\right) \times \sum_{a=1}^{A} (R_{I_a}/S_{I_a})$$

Finally, an OLS regression is applied with $\log(R/S)$ as the dependent variable and $\log(n)$ being the independent variable. The Hurst exponent, *H,* is obtained from the slope coefficient of the regression. An *H* of 0.5 means the series under investigation exhibits characteristics in line with the random walk hypothesis. An *H* greater than 0.5 denotes persistence while an *H* lower than 0.5 denotes anti-persistence.

Once *H* is computed, the autocorrelation within the time series is computed as:

$$CN = 2^{(2h-1)} - 1$$

According to Peters (1994), the *CN* represents the percentage of movements in the time series that can be explained by historical information. A *CN* = 0 signifies randomness in the time series under consideration pointing to a weak-form efficient market where historical information cannot be relied on to outperform the market.

4. Results and Discussion

Figure 1 shows the market capitalisation of the selected FTSE/JSE indices for the study. Figure 2 shows the statistical depiction of the data the study used.

Table 1: Market Capitalization of the Selected FTSE/JSE Indices

INDEX	MARKET CAPITALIZATION	DATE
FTSE/JSE TOP 40	R 8,283,699 MILLION	12 DECEMBER 2014
FTSE/JSE ALL SHARE	R 9,899,880 MILLION	12 DECEMBER 2014
FTSE/JSE SMALL CAP	R 306,991 MILLION	12 DECEMBER 2014

The kurtosis values for the indices selected are all larger than 3, which is the value for normal distribution signifying that all the series of the indices have fat tails compared to a normal distribution and leptokurtic. The returns of the indices therefore have frequent extremely large deviations from the mean with the FTSE/JSE Small Cap exhibiting the highest leptokurtosis.

Table 2: Summary Statistics for FTSE/JSE Indices

STATISTIC	FTSE/JSE All Share	FTSE/JSE Top 40	FTSE/JSE Small Cap
N	4840	4840	4840
Mean	0.000209	0.000203	0.000212
Standard deviation	0.005393	0.005934	0.002919
Skewness	-0.477473	-0.403190	-1.769811
Kurtosis	9.284673	9.290103	17.63013
Minimum	-0.055112	-0.062047	-0.033932
Maximum	0.032238	0.036685	0.017227
p – value*	0.000000	0.000000	0.000000
Anderson-Darling (A^2) Test	44.56388	41.43948	81.56400
p-value for A^2Test	0.0000	0.0000	0.0000

*Significant at 0.01 level

The series of all the indices are also negatively skewed, again with the FTSE/JSE Small Cap displaying the highest (in absolute terms) of negative skewness. The Anderson-Darling test also rejects the null hypothesis of a normal distribution at the 0.01 significance level. The implications of these findings are that the series of indices considered in this study show significant and frequent deviations from the mean, and therefore applying statistical models that do not take fatter tails into consideration will underestimate the likelihood of very good or very bad outcomes.

Table 3: BDS Test for FTSE/JSE Top 40

Dimension	BDS Statistic	Std. Error	z-Statistic	Prob
2	0.020296	0.001263	16.06869	0.0000
3	0.043829	0.002005	21.86348	0.0000
4	0.061754	0.002384	25.90133	0.0000
5	0.071797	0.002482	28.92776	0.0000
6	0.076036	0.002391	31.80620	0.0000
7	0.076134	0.002188	34.79629	0.0000
8	0.073571	0.001931	38.09098	0.0000
9	0.069521	0.001660	41.88956	0.0000
10	0.064655	0.001397	46.29486	0.0000

Table 4: BDS Test for FTSE/JSE All Share

Dimension	BDS Statistic	Std. Error	z-Statistic	Prob
2	0.020446	0.001272	16.07334	0.0000
3	0.044719	0.002019	22.15165	0.0000
4	0.063110	0.002401	26.28666	0.0000
5	0.073500	0.002499	29.40969	0.0000
6	0.077907	0.002407	32.36510	0.0000
7	0.078095	0.002203	35.44835	0.0000
8	0.075613	0.001945	38.88169	0.0000
9	0.071584	0.001671	42.84050	0.0000
10	0.066711	0.001406	47.44444	0.0000

BDS Test: The results for the BDS test on the three indices are presented in figures 3, 4 and 5. All BDS test statistics are presented at the 0.01 significance level. The BDS test is a robust statistical tool for differentiating non-linear stochastic systems or deterministic chaos from random independent and identically distributed systems.

Table 5: BDS Test for FTSE/JSE Small Cap

Dimension	BDS Statistic	Std. Error	z-Statistic	Prob
2	0.024080	0.001291	18.64708	0.0000
3	0.045668	0.002046	22.31723	0.0000
4	0.061074	0.002430	25.13436	0.0000
5	0.068026	0.002526	26.93400	0.0000
6	0.070060	0.002429	28.84341	0.0000
7	0.068195	0.002220	30.72152	0.0000
8	0.064567	0.001957	33.00075	0.0000
9	0.059558	0.001679	35.47940	0.0000
10	0.054089	0.001411	38.34702	0.0000

The series are examined up to 10 dimensions in line with Oppong, Mulholland and Fox (1999) and Bhattacharya & Sensarma (2006). The z—statistic is given as the BDS test divided by the standard error and is the final step that is used to test the null hypothesis. The null hypothesis of iid is not accepted if the z-statistic is greater than 2.58 at 0.01 level of significance. Given that the z-statistics are all greater than 2.58 for all the ten dimensions for the indices selected and p-values of 0.0000, the study concludes that the times series of returns for all the three indices do not exhibit randomness at 0.01 significance level.

Rescaled Range Analysis: Hypothetically, the H suggests some trading strategies, for example, H greater than 0.5 signifies persistence in the time series, and an H less than 5 signifies reversion to the mean, and H = 0.5 signifies randomness in the time series, therefore the more divergent the H, the less efficient the market is. Figures 6 and 7 present the outcome of the R/S analysis of the FTSE/JSE indices selected for the study.

Table 6: Average R/S for Each Value of *n*

N	FTSE/JSE Top 40	FTSE/JSE Small Cap	FTSE/JSE All Share
4840	69.63505	190.7382	73.78643
2420	53.53300766	138.1191622	56.1917233
1613	52.50887127	103.6487477	53.50605385
1210	42.71046899	82.26782126	44.12135614
968	34.4884864	63.74142505	35.87497224
605	28.48231733	44.88302406	29.2172694
484	25.71719924	41.59856064	27.10216567
242	18.36267392	28.20862672	19.21426979

Table 7: Hypothesis Test for *H*

	FTSE/JSE Top 40	FTSE/JSE Small Cap	FTSE/JSE All Share
$C = \exp(b_0)$	0.179338	-0.193979	0.189598
$H = b_1$	0.460994	0.679026	0.463352
CN	-0.052637863	0.281693572	-0.049535852
R^2	0.973949	0.987767	0.980835
SE (C)	0.093843	0.030850	0.080617
SE (H)	0.030779	0.030850	0.026442
*T-test (C)**	1.911052	-2.062362	2.351825
*T-test (H)**	14.97736	22.01089	17.52357
Prob (C)	0.1046	0.0848	0.0569
Prob (H)	0.0000	0.0000	0.0000

The FTSE/JSE Top 40 and the FTSE/JSE All Share exhibit slight reversion to the mean with an *H* of 0.461 and 04634, respectively. The correlation coefficients are -0.0526 and -0.0495 for the FTSE/JSE Top 40 and the FTSE/JSE All Share, respectively, implying that only 5.26% of the movements in the time series of the FTSE/JSE Top 40 are dependent on historical data and 4.95% for the FTSE/JSE All Share index. The FTSE/JSE Small Cap, however, displays significant persistence with an *H* of 0.6790 and a correlation coefficient of 0.2817, implying that 28.17% of movements in this index are dependent on historical information. Given that the FTSE/JSE All Share is a free-float market weighted index, the time series of its returns will be significantly influenced by the large caps companies and therefore the *H* for the series will be similar to that of the FTSE/JSE Top 40 as can be seen from figure 7. A high *H* according to Peters (1996), implies less risk, clearer trend and less noise and therefore the FTSE/JSE Small Cap can be construed to be less risky than the FTSE/JSE All Share and FTSE/JSE Top 40 contrary to the popular notion that small cap indices and stocks are riskier. Jefferis and Smith (2005) conclude that the JSE is weak form efficient. Peters (1996;18), however, posits that the efficient market hypothesis in its pure form does not accept only iid observations and does not necessarily entail independence over time, asserting that "if returns are random then the market is efficient. The converse may not be true, however." The study corroborates the conclusions of Smith (2008), that the JSE does not exhibit a random walk.

The findings of this study are in line with the assertion that small cap companies are less explored or totally ignored by many analysts and a large population of investors, and therefore the market for small stocks tend to be inefficient compared to their large cap counterparts, leading to prices deviating from fair values (Fundamental Index, 2008; Foley, 2014; Credit Suisse, 2014). Kuppor (2013) argues that small cap markets require less efficiency, otherwise this market that historically has created jobs, brought about break-through technologies while rewarding investors with price escalation will seize up for good. This finding further corroborates the assertions of McLean & Pontiff (2016) who argue that mispricing exist in financial markets and investors learn about these mispricing from academic publications. Financial markets can therefore not be construed to incorporate all relevant information since factor models purely reflect risk-return trade-offs and should not be affected by the publications done by academics. There are at least 316 factors that have been tested by financial market researchers that explain the cross-section of expected returns and many of the factors discovered are only significant by chance (Harvey, Liu and Zhu, 2015).

5. Conclusion

The study finds that the time series of returns of the JSE are not random. The FTSE/JSE Small Cap exhibits a high persistence while the FTSE/JSE All Share and the FTSE/JSE Top 40 exhibit slight mean reversion. Given that the JSE All Share is a free-float market cap-weighted index, the time series of its return will be heavily influenced by the large market cap companies, and therefore will exhibit characteristics similar to the FTSE/JSE Top 40. The study concludes that the FTSE/JSE Small Cap exhibits highly exploitable inefficiencies relative to the FTSE/JSE All Share and Top 40. As the small market cap companies are less popular, they will not be as highly researched by analysts and investors as their large cap counterparts, and therefore will exhibit exploitable inefficiencies. The study further concludes that the FTSE/JSE Small Cap exhibits less risk, less noise, clearer trend and more persistence, and therefore, contrary to the popular belief that small cap companies are riskier than large cap companies, at least on the JSE, the small cap index is less risky than the top 40 index and all share index as the H exponent of the FTSE/JSE Small Cap is significantly higher than 0.5 as compared to the FTSE/JSE Top 40 and all share index. This finding can be corroborated by the high standard deviation of 0.005934 for the FTSE/JSE Top 40 and 0.005393 for the FTSE/JSE All Share, as compared to 0.002919 for the FTSE/JSE Small Cap. In line with Peters (1996), we find an index with a higher H to be less risky than an index with a low H. This study therefore recommends a fractal approach to evaluating risk, as this provides a more adequate description of financial market behaviour. This paradigm would permit practitioners in financial and risk management to work with appropriate models to achieve their objectives as this would imply better analytical tools which can augment their awareness and understanding of the risk in financial markets. Table 8 presents the results of the linear regression of log N and log R/S.

Table 8: Result of linear Regression of Log N and Log R/S

ALL SHARE		
LOG N	**LOG R/S**	**SLOPE**
3.684845	1.867977	0.463352057
3.383815	1.749672	
3.207634	1.728403	**AUTOCORRELATION**
3.082785	1.644649	-0.049535852
2.985875	1.554792	
2.781755	1.46564	
2.684845	1.433004	
2.383815	1.283624	
TOP 40		
LOG N	**LOG R/S**	**SLOPE**
3.684845	1.842828	0.460993959
3.383815	1.728622	
3.207634	1.720233	**AUTOCORRELATION**
3.082785	1.630534	-0.052637863
2.985875	1.537674	
2.781755	1.454575	
2.684845	1.410224	

2.383815	1.263936	
SMALL CAP		
LOG N	**LOG R/S**	**SLOPE**
3.684845	2.280438	0.679025691
3.383815	2.140254	
3.207634	2.015564	**AUTOCORRELATION**
3.082785	1.91523	0.281693572
2.985875	1.804422	
2.781755	1.652082	
2.684845	1.619078	
2.383815	1.450382	

Table 9 presents the R/S values of all the sub samples used in the study.

Table 9: R/S Values for all Sub Samples

ALL SHARE							**TOP 40**					**SMALL CAP**		
N	**ST DEV**	**MAX – MIN**	**R/S**	**LOG R/S**	**n**	**ST DEV**	**MAX – MIN**	**R/S**	**LOG R/S**	**n**	**ST DEV**	**MAX - MIN**	**R/S**	**LOG R/S**
48	0.005	0.3978	73.786	1.867	48	0.002	0.5566	190.73	2.280	48	0.005	0.4131	69.63	1.842
40	3925	948	4305	9765	40	918	62	8196	438	40	933	46	5054	828
24	0.005	0.2834	54.531	1.736	24	0.003	0.4287	130.93	2.117	24	0.005	0.3038	52.72	1.722
20	1986	876	4570	6471	20	275	59	0872	042	20	762	06	5641	022
24	0.005	0.3227	57.851	1.762	24	0.002	0.3649	145.30	2.162	24	0.006	0.3314	54.34	1.735
20	5796	895	9896	3183	20	512	59	7452	288	20	099	27	0374	123
16	0.005	0.2887	53.255	1.726	16	0.003	0.3094	83.261	1.920	16	0.006	0.3138	52.10	1.716
13	4215	256	3927	3636	13	717	86	101	442	13	023	37	8335	907
16	0.005	0.2830	55.438	1.743	16	0.002	0.3502	144.02	2.158	16	0.005	0.3021	54.10	1.733
13	1052	258	8876	8145	13	432	53	8537	449	13	584	58	8139	263
16	0.005	0.2920	51.823	1.714	16	0.002	0.2006	83.656	1.922	16	0.006	0.3168	51.31	1.710
13	6361	851	8812	5299	13	398	01	605	500	13	175	24	0139	203
12	0.005	0.2683	48.773	1.688	12	0.003	0.2498	63.276	1.801	12	0.006	0.2859	46.37	1.666
10	5018	413	6630	1854	10	949	67	024	239	10	165	34	7784	310
12	0.004	0.2506	51.409	1.711	12	0.002	0.2354	97.588	1.989	12	0.005	0.2709	50.84	1.706
10	8764	922	0228	0393	10	412	17	900	400	10	328	26	8449	278
12	0.006	0.3318	49.219	1.692	12	0.002	0.3643	122.19	2.087	12	0.007	0.3398	46.26	1.665
10	7420	372	7585	1395	10	982	59	9737	070	10	346	84	8318	284
12	0.004	0.1110	27.082	1.432	12	0.001	0.0888	46.006	1.662	12	0.004	0.1236	27.34	1.436
10	0998	338	9802	6965	10	930	09	624	820	10	520	19	7324	915
96	0.005	0.2497	45.439	1.657	96	0.003	0.2578	64.631	1.810	96	0.006	0.2504	40.38	1.606
8	4959	274	2509	4312	8	989	10	356	443	8	202	63	7359	245
96	0.005	0.2052	38.361	1.583	96	0.002	0.1725	57.882	1.762	96	0.005	0.2278	39.18	1.593
8	3513	827	1762	8919	8	982	88	648	548	8	813	01	6015	131
96	0.004	0.1194	25.689	1.409	96	0.002	0.1047	46.942	1.671	96	0.005	0.1235	24.19	1.383
8	6492	366	7840	7605	8	231	31	549	567	8	106	64	9807	812
96	0.007	0.2822	40.222	1.604	96	0.003	0.2986	99.232	1.996	96	0.007	0.3050	39.86	1.600
8	0174	579	3625	4676	8	010	92	803	655	8	651	20	7454	618
96	0.003	0.1169	29.662	1.472	96	0.001	0.0921	50.017	1.699	96	0.004	0.1255	28.80	1.459
8	9437	786	2876	2046	8	843	69	770	124	8	359	56	1797	420
60	0.004	0.1229	28.500	1.454	60	0.002	0.1082	39.402	1.595	60	0.004	0.1427	28.59	1.456
5	3143	574	2061	8480	5	746	02	609	525	5	993	80	7642	330
60	0.006	0.2588	39.978	1.601	60	0.004	0.2407	49.525	1.694	60	0.007	0.2633	36.84	1.566
5	4746	460	4451	8259	5	860	08	216	826	5	147	52	7068	403
60	0.005	0.1844	34.844	1.542	60	0.002	0.1001	36.201	1.558	60	0.005	0.1967	34.41	1.536
5	2941	669	1750	1302	5	766	17	640	728	5	718	88	3215	725
60	0.004	0.1567	35.432	1.549	60	0.001	0.0975	50.153	1.700	60	0.004	0.1668	33.97	1.531
5	4225	023	9502	4073	5	944	03	757	303	5	911	51	6960	185
60	0.005	0.1201	23.547	1.371	60	0.002	0.1132	43.586	1.639	60	0.005	0.1224	22.07	1.343

5	1039	853	7100	9487	5	599	70	294	350	5	550	92	1010	822
60	0.008	0.2740	34.060	1.532	60	0.003	0.2428	73.978	1.869	60	0.008	0.2978	33.94	1.530
5	0465	678	3709	2494	5	283	67	970	108	5	776	88	4275	767
60	0.004	0.0829	17.851	1.251	60	0.002	0.0733	36.170	1.558	60	0.005	0.0940	18.29	1.262
5	6456	325	9202	6849	5	027	20	195	351	5	142	77	5484	344
60	0.003	0.0676	19.522	1.290	60	0.001	0.0548	30.045	1.477	60	0.003	0.0748	19.71	1.294
5	4676	965	3779	5327	5	824	17	512	780	5	796	34	2885	750
48	0.002	0.0984	34.042	1.532	48	0.001	0.1139	65.174	1.814	48	0.003	0.0981	28.37	1.453
4	8919	486	9441	0271	4	749	95	859	080	4	458	43	9391	003
48	0.007	0.2342	32.488	1.511	48	0.005	0.2433	45.427	1.657	48	0.008	0.2335	28.99	1.462
4	2110	712	2047	7257	4	358	91	205	316	4	056	57	0108	250
48	0.005	0.1639	31.862	1.503	48	0.003	0.1308	39.365	1.595	48	0.005	0.1744	31.34	1.496
4	1465	777	1358	2749	4	323	17	244	113	4	567	79	2110	128
48	0.005	0.2260	40.783	1.610	48	0.002	0.0695	26.943	1.430	48	0.006	0.2455	40.64	1.609
4	5415	034	9233	4890	4	581	33	618	456	4	040	12	9137	051
48	0.004	0.1076	25.862	1.412	48	0.001	0.0790	41.946	1.622	48	0.004	0.1099	23.76	1.375
4	1634	754	2603	6665	4	884	11	082	691	4	625	21	4678	932
48	0.005	0.0947	18.619	1.269	48	0.002	0.0979	38.694	1.587	48	0.005	0.0970	17.49	1.243
4	0882	395	6163	9707	4	530	01	780	652	4	544	21	9438	024
48	0.008	0.2385	29.244	1.466	48	0.003	0.1919	55.923	1.747	48	0.008	0.2611	29.38	1.468
4	1582	828	4117	0429	4	432	27	059	591	4	885	10	6872	153
48	0.005	0.1035	18.390	1.264	48	0.002	0.0871	35.141	1.545	48	0.006	0.1085	17.64	1.246
4	6302	413	3333	5896	4	481	78	627	822	4	155	83	0457	510
48	0.004	0.0844	19.711	1.294	48	0.001	0.0723	40.858	1.611	48	0.004	0.0923	19.32	1.286
4	2865	934	6611	7232	4	770	23	631	284	4	781	80	1856	049
48	0.003	0.0714	20.016	1.301	48	0.001	0.0506	26.510	1.423	48	0.003	0.0785	20.19	1.305
4	5674	060	1661	3809	4	909	14	502	418	4	891	96	7946	307
24	0.002	0.0580	19.750	1.295	24	0.001	0.0739	46.747	1.669	24	0.003	0.0572	16.11	1.207
2	9407	807	6103	5805	2	581	29	334	757	2	551	27	3776	197
24	0.002	0.0515	18.196	1.259	24	0.001	0.0594	31.329	1.495	24	0.003	0.0551	16.45	1.216
2	8323	373	5086	9881	2	899	80	850	958	2	351	51	7386	361
24	0.006	0.1813	26.173	1.417	24	0.004	0.1963	40.358	1.605	24	0.007	0.1945	24.80	1.394
2	9271	053	2330	8574	2	866	89	023	930	2	845	78	1812	483
24	0.007	0.2078	27.774	1.443	24	0.005	0.2116	36.575	1.563	24	0.008	0.1991	24.10	1.382
2	4833	417	0440	6391	2	787	51	105	186	2	262	80	7543	153
24	0.005	0.1645	29.793	1.474	24	0.003	0.1353	35.990	1.556	24	0.006	0.1751	29.12	1.464
2	5223	281	1821	1169	2	761	65	409	187	2	013	45	8148	313
24	0.004	0.0768	16.214	1.209	24	0.002	0.0620	22.027	1.342	24	0.005	0.0817	16.08	1.206
2	7409	728	8545	9131	2	817	48	845	972	2	082	32	3000	367
24	0.005	0.1604	27.752	1.443	24	0.002	0.0675	23.717	1.375	24	0.006	0.1765	28.13	1.449
2	7818	602	5211	3024	2	850	96	819	075	2	275	33	4392	238
24	0.005	0.1044	19.969	1.300	24	0.002	0.0457	20.066	1.302	24	0.005	0.1104	19.27	1.284
2	2318	761	6274	3700	2	279	34	204	465	2	732	59	0597	895
24	0.004	0.0737	16.307	1.212	24	0.002	0.0506	25.141	1.400	24	0.005	0.0775	15.50	1.190
2	5208	210	1162	3772	2	016	88	075	384	2	005	92	1646	378
24	0.003	0.0743	19.723	1.294	24	0.001	0.0653	37.629	1.575	24	0.004	0.0773	18.36	1.264
2	7707	714	4325	9825	2	737	67	866	533	2	210	18	6903	036
24	0.004	0.0771	18.530	1.267	24	0.002	0.0437	20.916	1.320	24	0.004	0.0812	17.84	1.251
2	1625	310	0827	8774	2	092	53	857	496	2	552	43	5919	539
24	0.005	0.0919	15.669	1.195	24	0.002	0.0913	31.518	1.498	24	0.006	0.0928	14.54	1.162
2	8687	579	1700	0460	2	898	48	543	566	2	383	73	9943	861
24	0.006	0.1090	18.166	1.259	24	0.003	0.0987	31.864	1.503	24	0.006	0.1144	17.53	1.243
2	0011	205	6638	2752	2	100	89	967	313	2	529	60	2285	839
24	0.009	0.1870	19.014	1.279	24	0.003	0.0902	24.373	1.386	24	0.010	0.2039	19.02	1.279
2	8375	536	3923	0824	2	702	37	617	920	2	718	52	8976	415
24	0.006	0.0895	13.908	1.143	24	0.002	0.0768	28.532	1.455	24	0.007	0.0940	13.36	1.125
2	4420	998	7078	2868	2	693	46	931	346	2	038	52	3327	915
24	0.004	0.0701	15.007	1.176	24	0.002	0.0540	24.063	1.381	24	0.005	0.0741	14.50	1.161
2	6721	171	6342	3122	2	246	52	185	353	2	114	96	8619	626
24	0.005	0.0652	12.641	1.101	24	0.001	0.0453	22.859	1.359	24	0.005	0.0718	12.50	1.097
2	1598	270	4330	7963	2	983	42	828	073	2	747	63	3739	040
24	0.003	0.0484	15.272	1.183	24	0.001	0.0317	21.100	1.324	24	0.003	0.0567	15.98	1.203
2	1723	491	4674	9092	2	503	06	377	290	2	553	82	0517	591
24	0.003	0.0581	15.005	1.176	24	0.001	0.0396	20.566	1.313	24	0.004	0.0604	14.29	1.155
2	8730	168	6835	2558	2	929	69	036	151	2	231	61	0721	054

24 2	0.003 2326	0.0627 583	19.414 0312	1.288 1157	24 2	0.001 885	0.0354 32	18.792 664	1.273 988	24 2	0.003 519	0.0692 62	19.68 4227	1.294 118

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