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The Need to go Beyond Deterministic Data Envelopment Analysis (DEA): A Comparative Analysis with Bootstrapping DEA in Risk Management Efficiency Measurements

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Abstract: This paper attempts to rectify the flaws of deterministic data envelopment analysis (DEA), induced from the characteristics of its approach. The disadvantages have drawn considerable attention in the area of banking efficiency studies, and bootstrapping DEA (BDEA) analysis has been proposed in the literature. This paper provides a comparative analysis of each approach to understand their theoretical foundations and mathematical computations, as well as their strengths and weaknesses. The arguments were empirically supported within the context of risk management efficiency analysis in banking institutions. Finally, the paper proved the need for BDEA analysis to be used while advancing measurement precision for risk management efficiency measurements.

Keywords: *Data Envelopment Analysis, Bootstrapping DEA, Deterministic, Efficiency, Banking Institutions.*

1. Introduction and Background

Data Envelopment Analysis (DEA) has been extensively researched in the fields of science and social science studies. In the vast literature concerning issues in efficiency studies, an extension to the deterministic underscores the accuracy and obtaining more reliable efficiency scores. From a practical point of view, the bootstrapping DEA (BDEA) approach enables to purify of DEA scores from the concept of artificial efficiency, thus minimizing the estimation problems in deterministic measurement. Moreover, the original DEA evaluation assumes that the input and output parameters used in the model estimation translate into perfect measurement of efficiency. DEA is a purely deterministic analytical method and does not contain a stochastic term in its optimized linear programming approach (Filippou and Zervopoulos 2011). By highlighting the uncertainty issue in data, this study provides an alternative methodological approach realizing that the linear programming approach strongly pushes the analysis with a true situation which is never the same as data gathered that has been addressed.

Due to its nature as a deterministic approach, the DEA model of efficiency ignores the fact that all the input and output variables are not deterministic (Simar and Wilson 2008; Sengupta 2000) and it does not account for the measurement of errors and random factor in the data (Al-Rashidi, 2016). Zhang and Bartels (1998) also stated that the estimated mean value of the technical efficiencies was dependent on the parameter values. Banker (1993) verified weak consistency of the DEA estimation particularly when the measurement involves a single input and single output case. As a result, they stressed the need for a stochastic frontier analysis to correct DEA's approach to efficiency measurement. The present work aims to highlight the weakness of the efficiency score obtained by a deterministic DEA approach; particularly when the DEA approach ignores the risk management context required to produce a trustworthy assessment of a firm's risk management efficiency model. The stochastic model is transformed to the DEA deterministic model which will become the basis for the theoretical development of this model.

This paper is organized as follows: Section 2 reviews the stochastic BDEA approach and the need to go beyond the deterministic DEA estimation. This is followed by an introduction to bootstrapping DEA method in the context of risk analysis and management. In Section 4, the bootstrapping approach in risk management efficiency measurement is discussed, followed by the theoretical approach of bootstrapping technique for optimization of DEA in Section 5. Section 6 provides the model and formulation of BDEA. Next, Section 7 provides the comparison of findings between the DEA and BDEA approaches. Finally, Section 8 concludes the paper.

2. Literature Review – The Need to Go Beyond Deterministic DEA Estimation

In this context, Cummins et al. (2003) and Simar and Wilson (1998) believe that DEA can create bias efficiency estimation as it refers to upward bias in finite samples in its evaluation. When the relative measurement of the best practice is observed in the sample, it can create a biased estimator (Maghyereh and Awartani 2012). As the scores in DEA are calculated through an unknown Data Generating Process (DGP) (Xue and Harker 1999) without any confidence interval in the estimation, the results will be dependent on each other, thus creating a dependency problem. The sampling distribution error along with the asymptotic bias, will affect the DEA estimation as the scores generated are less sensitive towards the sampling variation (Gijbels et al., 1999). In addition, the deterministic frontier is a non-statistical method that does not account for any random factor in the data, such as random noise or measurement errors, and it is estimated either by implementing mathematical programming or by means of econometric regression techniques (Al-Rashidi, 2016; Murillo-Zmorano, 2004; Jacobs, 2001).

Verification test statistics have very poor power in small samples, so it does not appear possible for a bank or its supervisor to verify the accuracy of the DEA approach in efficiency estimation unless several years of performance data are available. For instance, Zhang and Bartels (1998) investigated the sample size-bias effect on the mean productive efficiency of electricity distribution industries in Australia, New Zealand and Sweden. The results indicated that an increase in the number of firms reduced the mean of the overall technical efficiency (OTE) of the industry. Using a Monte Carlo simulation on the DEA, Zhang and Bartels (1998) confirm that when the number of observations was added to the sample, more structural inefficiency was observed. This shows that sample size bias leads to potentially wrong interpretations (biased estimation of performance measurement and efficiency estimation).

This discussion highlights the weakness of the efficiency score obtained by a deterministic DEA approach. One particular model for accuracy under uncertainty is called a stochastic DEA model. The stochastic model in DEA is for the possibility of variations in input and output parameters where the efficiency measure of a DMU is defined via joint probabilistic comparisons of inputs and outputs with other DMUs (Huang and Li 2001). Under the stochastic model, Brázdik (2004) emphasizes that the derivation of a nearly 100 percent (1.000) confidence chance-constrained problem is revised. It is necessary to estimate the efficiency score effectively to enhance decision-making and benchmarking strategies (Moradi-Motlagh et al., 2014).

Bootstrapping for Risk Analysis and Management: The bootstrapping model consistently applies the expert's rules to test whether the decisions and predictions from the model are similar to those from the experts (Armstrong 2001). Even though Armstrong uses the judgmental bootstrapping model to evaluate experts' predictions, Allen and Fildes (2001) state that bootstrapping is an accepted procedure in the social sciences and econometrics to provide more realistic outcomes and solutions. The bootstrapping procedure in statistical analysis is required to produce a reliable assessment of a risk management model's accuracy in predicting the efficiency measurement of the firms. Zenti and Pallotta (2000) note that the bootstrapping approach can provide satisfactory assessments of strategic risk. Whether decision-making can be determined as "accepted" or "tolerable" based on risk analysis, the decision must proceed by the accuracy of the assigning and assessing method.

In the banking sector, bootstrapping has been used alongside expert advice since the 1970s. Slovic et al. (1972) propose the bootstrapping approach to examine the security return performance based on specific risk, and Ebert and Kruse (1978) also revealed superior performance in the field of security analysis by developing bootstrapping models for five analysts who forecasted returns for 20 securities using information on 22 variables and its associated risks. Given that the models violated guidelines for developing bootstrapping models, it is surprising that the bootstrapping models were more accurate than analysts for 72% of the comparisons of management capabilities assessment. Previous scholars like Abdel-Khalik and El-Sheshai (1980) use the bootstrapping model to investigate actual default in loans in comparison with figures predicted by 28 commercial bank lending officers. They found that the prediction model by bootstrapping models was as accurate as the prediction of all officers. This bootstrapping analysis before awarding loans helped the banks to evaluate the risk from potential defaults to reduce costs and the likelihood of bias in awarding loans.

Another risk component that has been discussed by Libby (1976) concerns the prediction of bankruptcy for 60 large industrial corporations. But this study in contrast to those illustrated earlier showed that the bootstrapping model used in insolvency risk and its relation to firms' bankruptcy is less accurate compared to experts' opinions and decisions due to both quantitative and qualitative analyses used in the study. However, Goldberg (1976) questions the validity of Libby's results as the analysis suffered from severe skewness in the causal variables and re-analyzed the data. He found that the percentage of times produced by the bootstrapping model beat the expert opinion and increased its accuracy by 49% (from 23% to 72%); which indicates a huge difference in efficiency estimations. Therefore, the bootstrapping model that has been applied to evaluate firms' bankruptcy due to related risk elements is more reliable for risk management evaluation and estimation. In addition to these studies that are guided by conventional financial aspects of efficiency, Chortareas et al. (2013) construct an interesting application of the bootstrapping technique in a more abstract matter of banking efficiency relating to economic freedom. They employed a robust bootstrap procedure to regress the first-stage efficiency scores on economic freedom indices in banks operating in 27 European Union member countries. The variable of economic freedom was proxied by six governance indicators representing government quality that were replicated by 2000 bootstrap replications. Extending bootstrapping to a new area of efficiency analyses this study again highlights the superiority of bootstrapping in accurate measurement of efficiency in financial institutions.

In short, the literature presented in this subsection indicates the ability of the bootstrapping approach in the risk analysis context. Empirical results consistently show that bootstrapping models are associated with increased accuracy in measuring risk elements. The results from the bootstrapping model have always proved to be more accurate than those of expert analysts. Though the relationship between bootstrapping and the risk management context has not been developed theoretically, empirical evidence has found that the principle of the bootstrapping model is able to deliver analyses of specific risks in the banking sector with reliability and accuracy.

In relation to the need for bootstrapping elements to be considered while risk management issues were raised; the bootstrapping technique as an extension to the traditional DEA measurement is pivotal to enhance traditional measurement of firms' efficiency measurement. The study applies the bootstrapping technique to simulated risk management efficiency, which allows for reliable calculations. This method provides a flexible, robust, intuitive and comprehensive risk management efficiency evaluation. To this extent, the focus has been on the relationship between risk management efficiency measures with bootstrapping techniques. For a clearer picture, the example that can be used in this paper is when financial instrument tools such as financial derivative instrument is used for risk management purposes. In banking sectors, while they are facing huge uncertainties in their operations, the accuracy of analysis is highly significant to be performed. It generates financial scenarios over the derivative activity horizon in banking or financial institutions using the information based on derivative usage in hedging activities to minimize financial market risks and the need for multivariate empirical analysis. The usage of the bootstrapping approach extended from the DEA's traditional approach is suitable for addressing the implications of banking performance while facing uncertain environments. Table 1 provides evidence, which employed bootstrapping analysis in deterministic DEA measurements in banking efficiency studies.

Table 1: Summary of Literature Considering Bootstrapping DEA Approach in Banking Environments

ARTICLE	BDEA APPROACH	RESULTS / CONCLUSION
Antunes, J., Hadi-Vencheh, A., Jamshidi, A., Tan, Y., & Wanke, P. (2024). Cost efficiency of Chinese banks: Evidence from DEA and MLP-SSRP analysis. Expert Systems with Applications, 237, 121432.	The study, first, introduces an innovative Data Envelopment Analysis (DEA) model to evaluate the cost efficiency of Chinese banks. Second, it proposes a Stochastic Structural Relationship Programming (SSRP) Model based on neural networks.	The research findings reveal that Chinese commercial banks gradually improved their efficiency from 2010 to 2015, experienced some volatility thereafter, and ended up with an efficiency score of 0.746 out of 1 by the end of 2018. The study also suggests that banks with lower efficiency levels benefit from improved efficiency, leading to increased profitability and a focus on traditional banking

		activities. Conversely, banks with higher efficiency levels should seek alternative, profitable banking ventures to maintain their efficiency.
Tsolas, I. E. (2021). Firm Credit Scoring: A Series Two-Stage DEA Bootstrapped Approach. *Journal of Risk and Financial Management*, *14*(5), 214.	The study employed two-stage data envelopment analysis (DEA) combined with bootstrap and hierarchical clustering.	A bootstrapped DEA-based synthetic indicator is developed to be used with the other performance metrics as inputs to hierarchical clustering to divide sample firms into credit risk clusters. Here, the bootstrapped approach used in this study could aid firms in evaluating their performance and increasing their competitive advantages.
Dia, M., Golmohammadi, A., & Takouda, P. M. (2020). Relative Efficiency of Canadian Banks: A Three-Stage Network Bootstrap DEA. *Journal of Risk and Financial Management*, *13*(4), 68.	The study proposes a novel three-stage (production, investment, and revenue generation) network Data Envelopment Analysis (DEA) with bootstrapping to evaluate the performance of the six big Canadian banks for the period 2000–2017, amid the 2007 financial crisis and the increasing competition level due to new technologies.	Authors have identified the best practices in each stage that can be used as benchmarks by other banks to improve their economic sustainability especially when dealing with financial crisis issues which refers to uncertainty in banking environments. They found that DEA provides more insightful and accurate results in terms of banks' efficiencies.
Khan, I. U., Ali, S., & Khan, H. N. (2018). Market concentration, risk-taking, and efficiency of commercial banks in Pakistan: An application of the two-stage double bootstrap DEA. *Business and Economic Review*, *10*(2), 65-95.	Following Simar and Wilson (2007), the study applies two-stage data envelopment analysis (DEA) with double bootstrapping in the analysis of Pakistan banking institutions for the period 2007 to 2014. With the concentration of uncertainty elements focusing on market concentration, capital risk, credit risk, and liquidity risk, these risks underscore the correlations to the need to use the bootstrapping approach.	Theoretically, the use of bootstrapping DEA analysis within the context of risks enriches the existing literature on banking with new insights. Methodologically, the DEA double bootstrapping procedure allows researchers to evaluate the impact of contextual variables (risk assessment within the context of banking institutions) on the performance of different types of DMUs. Thus, the consideration of DEA with two-stage double bootstrapping advances the understanding of bank efficiency with both systematic and unsystematic risk variables.
Stewart, C., Matousek, R. & Nguyen, T. N. (2016). Efficiency in the Vietnamese banking system: A DEA double bootstrap approach. *Research in International Business and Finance*, *36*, 96-111.	Using banking institutions in Vietnam as DMUs, the study employed efficiency measurement introduced by Simar and Wilson (2007) to explore the determinants of bank efficiency.	The results indicate that the use of bootstrapping elements in DEA measurements could influence Vietnamese banking efficiency and performance affected by the environment and the global financial crisis.
Diler, M. (2011). Efficiency, Productivity and Risk Analysis in Turkish Banks: A Bootstrap	By addressing the impacts of the 2007 global financial crisis on the efficiency and productivity of	The findings extended the existing DEA literature by applying the bootstrapping method to improve

DEA Approach. *Journal of BRSA Banking & Financial Markets*, *5*(2).	Turkish banks from the year 2003 to 2010, the issue of the existence of inherent dependency among DEA efficiency scores with the basic assumption of regression analysis exists. For instance, independence within the sample is violated. Therefore, to eliminate the dependency problem and to be able to make valid statistical inferences, the bootstrapping method is applied.	DEA efficiency and productivity estimates, particularly when observing the impacts of the recent 2007 global financial crisis.

Having examined the relationship between the bootstrapping approach in risk management measurement, it is now necessary to consider the application of bootstrap to ensure the reliability and accuracy of risk management efficiency results (Zakaria & Islam, 2019) and the test model effect the risk management efficiency measurement in banks in an influential way.

Bootstrapping Approach in Risk Management Efficiency Measurement: Most previous studies measure and manage different risk sources using parametric methods (e.g. Medova and Smith 2005; Gil and Polyakov 2003), whereas bootstrapping is able to measure specific risks. For instance, credit risk and default risk, which deal with Credit Metrics, only depend on the rating category without any contribution from idiosyncratic components and are seldom updated and are not sensitive to current market conditions. Marsala et al. (2004) strongly agree that the bootstrapping technique constructs a respected measurement analysis for obtaining a precise description of the risk management measurements. Focusing on one or more types of risk management products, the financial instrument is modelled robustly with a filtered bootstrap approach. In their study, they have applied a filtered bootstrap technique, as they believe that this non-parametric approach is able to simulate the stochastic processes and modelling risk factors. Another study by Hashemi et al. (2013) evaluated the risk assessment process using the bootstrapping approach to solve the issue of non-accuracy in port projects. By applying the bootstrap method, they have structured risk management approach in three phases; the first phase focused on assessing risk issues, providing risk handling plans and monitoring processes; the second phase involved project risk identification where risks are categorized into a specific structure; while the last phase is project risk assessment where non-parametric bootstrap method (B= Mi) is used to obtain compromise final risk ranking. From the findings, the study concludes that the bootstrap confidence interval approach can be applied to risk assessment problems yielding reliable and meaningful results that cannot often be obtained by the traditional approach.

In addition, Huang et al. (2009) have further extended the utility of bootstrapping in the risk management context and proved that the bootstrapping technique can simulate the distribution of future movements in external risk, which can be used as a forecasting tool. This analysis highlighted the remarkable ability of the bootstrapping approach, which not only functions as a bias-corrected technique but is also able to forecast future risk management strategies of the firm. Recently, Antunes et al. (2024) found that using a Stochastic Structural Relationship Programming (SSRP) Model based on neural networks to measure bank efficiency offers numerous advantages. These include the ability to capture nonlinear relationships, high accuracy, adaptability to changing environments, comprehensive multidimensional analysis, data integration, robustness to data variations, real-time assessment, granular insights, risk identification, customization, decision support, longitudinal analysis, benchmarking, and regulatory compliance. However, rigorous data preparation, validation, and model interpretation are crucial to ensure reliable and actionable results for bank management and regulatory purposes.

In short, the bootstrapping technique has several advantages. Before the methodological implications of integrating the bootstrap approach to DEA, this section has not only reviewed the limitations of DEA but has also established the relationship between the bootstrapping approach and risk management context, which has been neglected in most studies until recently. All this has highlighted the need to bring bootstrapping into dialogue with traditional DEA employed in this study as this study is concerned with generating accurate and reliable measurements of risk management efficiency.

3. Bootstrapping Technique for Stochastic Optimization of DEA – Theoretical Approach

Efficiency Bias Correction, Confidence Intervals Construction and Dependency Issue Incorporating Noise in Data and Sampling Error Issue: This study looks at the implication of using the financial derivative instrument in the context of risk management measurements; hence shows the need for multivariate empirical analysis. The usage of normal bootstrapping extended from the DEA traditional approach is suitable for the raw effect of financial derivatives because of the "volatility" condition on the risk financial market. This basic idea therefore drives the study to apply the BDEA formulations as stochastic amendments to the traditional DEA that is able to address the effect of noise in the data and the sampling error on efficiency estimation. BDEA overcomes a major weakness of DEA, which is its failure to deal effectively with the stochastic element of efficiency estimation (Hawdon 2003). This is because the observed data (input and output) would normally be subjected to the measurement error that can create noise in the data due to omitted input or output variables. Also, there may be events that affect the level of input-output of some of the production units. As in Stochastic Frontier Analysis (SFA), specific assumptions are made in BDEA for evaluating the distribution of the inefficiency to isolate the noise from efficiency.

In addition, Mellenbergh et al. (2008) state that bootstrapping was strongly recommended when the theoretical distribution of the statistics of interest is complex or unknown. Bootstrapping the DEA allows for the estimation of the sample distribution of almost any statistic using a simple method (Munisamy and Danxia 2013; Kao and Liu 2014). Since this procedure is distribution-independent it provides an indirect way of assessing both the actual distribution underlying the sample and the extent of interest derived from this distribution. Thus, the DEA bootstrap can be applied to encounter the intrinsic problems of measurement error in the standard DEA to estimate the bias-corrected DEA efficiency scores (Halkos and Tzeremes 2013; Halkos et al. 2012). Maghyereh and Awartani (2012) state that the DEA estimator tends to become biased when it measures relative efficiency from the best practice observations in the sample. DEA does not provide any assumptions regarding the exogenous factors or measurement error and does not allow confidence intervals. In this type of measurement, DEA may introduce an upward bias in measuring the efficiency scores as it depends on the best practice observations (Barros et al. 2014; Gutiérrez et al. 2014). In contrast, bootstrapping the DEA will correct the efficiencies for bias and estimate confidence intervals for them. This approach was chosen because it can be applied to the institutional efficiency measurement obtaining confidence intervals and bias-corrected efficiency.

In addition, Staat (2001) re-analyzed the original DEA data from Banker and Morey (1986) to conclude that the results were significantly influenced by the number of observations and sample size effect. In an early study on this matter, Efron (1979) introduced a bootstrapping technique as an alternative method of conducting inference when the sample size is small. Following that, Banker (1993) also shows the utility of bootstrap techniques as an alternative method for conducting inferences, particularly with a sample. DEA efficiency scores can also violate the inherent independence within the sample (Casu and Molyneux 2003). Xue and Harker (1999) were perhaps the first to address the problem of inherent dependency on efficiency scores when applied to the regression model. They explained that a number of studies used regression analysis (two-stage DEA) such as Tobit and Ordinary Least Square (OLS) to explain the variation of efficiency scores among different firms. The authors found that the efficiency results produced by the DEA approach were dependent on each other and independent within the sample because the DEA efficiency measured the relative efficiency index, not an absolute efficiency index. Bootstrapping DEA can solve dependency problems with the assumption that "the distances $QDEA - Q*$ are distributed as the distances $Q - QDEA$" (Fried et al. 2008, p.456). Problems can also arise in sampling distributions that are analytically intractable (Alonso et al. 2006). Alonso et al. (2006) also agreed that the bootstrapping approach was able to solve sampling distributions that are hard to trace due to pretesting and nonlinearity. Hence, they proposed a "smoothed bootstrap" procedure to improve the estimation and avoid remarking problems. This study has followed Atkinson and Wilson (1995) and Simar and Wilson (2000) who applied a smoothed distribution of efficiency values to the generation.

Motivated by these issues that highlight the limitations of the traditional DEA approach, and previous studies that have made improvements to it, this study uses the bootstrap DEA proposed by Simar and Wilson (1998, 2000) as well as Sadjadi and Omrani (2010) who developed a bootstrapping measurement for efficiency studies known as Bootstrapped DEA (BDEA). Based on the literature, this study believes that the BDEA's integration approach will strengthen the DEA and yield more accurate efficiency scores by addressing the following issues:

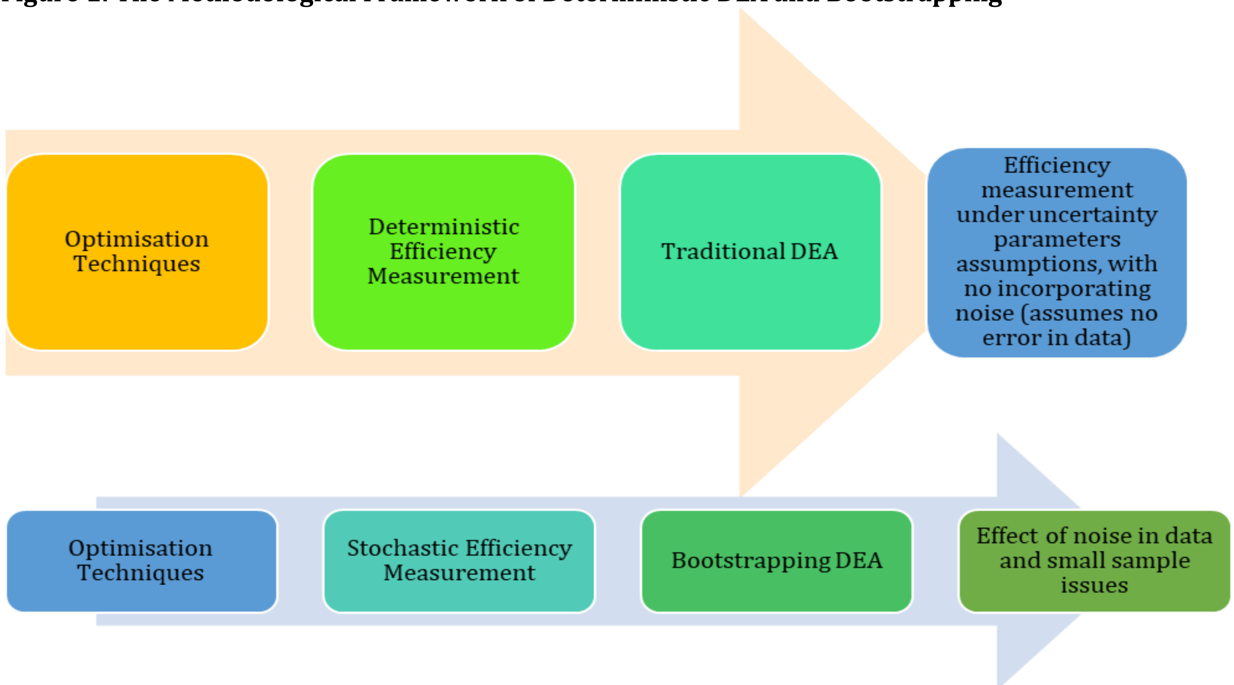
- BDEA can resolve the bias in the parameter estimates, thus, producing more reliable estimations for risk management efficiency (Simar and Wilson 2000; Assaf and Agbola 2011; Johnes 2006).
- BDEA can produce more reliable results in measuring risk management efficiency in banks (Zakaria et al. 2014).
- The bootstrap method predicts the future performance of the decision-making unit (DMU such as firms and organizations) which generally improves both the precision and the reliability of predictions (Johanson 2013; Friedman and Friedman 2013).
- Policy makers in banks will be provided with more trustworthy and dependable findings for them to strategies their risk management plan.

Bootstrap DEA in Banking Efficiency Studies: In 1997, Ferrier and Hirschberg examined the efficiency of Italian banks with the application of the bootstrapping method. The authors adapted the bootstrap technique in the context of the linear programming approach because of its computational power in measuring unknown values in true frontier production functions. They showed how bootstrapping can be used to obtain a sampling distribution of the efficiency scores of individual banks from which confidence intervals and a measure of bias can then be constructed. Ferrier and Hirschberg believe that the results generated by the bootstrapping approach allow the decision-maker to consider the reliability of the calculated efficiency scores more rigorously.

With a similar objective and methodology, Assaf et al. (2011b) evaluated the real values of technical efficiency in Saudi banks using a bootstrapping technique by resampling the original data. They found that the efficiency bias-corrected estimations for every observation were within the confidence interval. They consider the result derived from this analysis to be a statistical advantage over traditional DEA measurement as it used more rigorous criteria for evaluation. The same findings by Zakaria et al. (2014) compared the efficiency of twelve Islamic banks in Malaysia, in which the BDEA approach has been employed to resolve the uncertainty issue of traditional DEA measurement practice. They believe that adopting the BDEA approach is not only essential for a more precise efficiency measurement but also compatible with Islamic banking concepts that differ substantially in objectives and operations from those of conventional banks. The purpose of the BDEA approach is to reduce uncertainties suits Islamic banking principles to avoid any uncertainties in their operations.

In a different study, Assaf et al. (2011a) applied the bootstrap Malmquist index to provide a quantitative measure of productivity (total factor of productivity – TFP) change in Shinkin banks operating in Japan. With further support to the Malmquist results, the study found that efficiency productivity growth based on market share on deposit (MSD), number of branches, ROA, net interest margin (NIM) and deposit concentration ratio, are statistically significant to the efficiency scores except NIM. Based on the comparative results of normal efficiency estimates, they conclude that the bootstrap approach was able to provide less biased scores by correcting the efficiency estimates sensitivity to random variations in the data.

Figure 1: The Methodological Framework of Deterministic DEA and Bootstrapping



4. Bootstrap Data Envelopment Analysis: Model and Formulation

This study addresses the limitation of the DEA approach and obtains a non-parametric envelopment estimator of the DEA efficiency score applying the bootstrapping approach developed by Simar (1992) and Simar and Wilson (1998, 2000) who were pioneers in using the bootstrap in frontier models. This approach basically estimates a true sampling distribution by mimicking the data-generating process (DGP) by re-sampling the DEA data. The resampled data consists of original sample values using a selection of thousands of "pseudo samples" from the observed set of sample data. Repeating this process enables us to build a good approximation of the true distribution. However, Simar and Wilson (1998) retracted this method in the cases of nonparametric frontier estimation, arguing that the "pseudo sample" provided an inconsistent bootstrap estimation of the confidence intervals (the so-called naive bootstrap) since the distance estimation values are close to unity when directly re-sampled from the set of original data. It shows that the consistent information of such confidence intervals is closely dependent on the consistent replication of a DGP. Besides, the DEA estimator may produce a large number of apparently efficient units with $\hat{\theta} = 1$ (the number of such units is likely to increase with p, the number of inputs). It will consequently influence $\hat{F}$ (density function) which will provide a poor estimation of Fnear the upper boundary. In particular, bootstrap estimates may be inconsistent if this issue is not raised.

The bootstrap algorithm starts with some basic definitions:

- A production set is defined as$\psi = \left\{(x,y)\epsilon R_{+}^{p+q} \middle| x\ can\ produce\ y\right\}$, where the amount of some p inputs $x$that can produce $q$outputs$y$, while the set of inputs that make the output level y possible is defined as. $$X(y) = \left\{x\epsilon R_{+}^{p+q} \middle| (x,y)\epsilon\ \psi\right\}$$
- The efficient production limit can be defined as the subset of(y) such that $\partial(y)$ = $\{x|x \in X(y), \theta x \notin Xy \forall \theta \epsilon (0,1)\}$ where it describes the possibility to obtain more outputs with a given level of input. The sets ofψ, (y), θ_1 and (y) are unknown, meaning that, if we assume that some DGP,, generates a random sample$X = \{x_i y_i | i = 1, \ldots, n\}$ of $n$homogenous organizations (firms).
- Specifically,$\hat{\theta}_i$ can be obtained by DEA application which indicates that the firm is completely technically efficient. Input orientation has been chosen and defined as $\theta_i = \{\theta | \theta x_1 \in (y)\}$, where it explains the DEA input-orientated efficiency measurements. $\theta_i = 1$ indicates that the input and output unit(x_i, y_i) is fully efficient (100%).

Since DEA estimates a production frontier boundary, generating bootstrap samples is not a straightforward process. This approach is based on the DEA estimator by drawing replacement from the original estimates of theta, $\theta_{(x,y)}$ and then applying the reflection method proposed by Silverman (1986). As the efficiency measures being considered in this study are input-based, the bootstrap is performed over the original risk management efficiency scores. The steps for a smoothed bootstrap algorithm can be summarized as follows:

- Compute the original DEA model to obtain efficiency scores $\hat{\theta}_1 \dots \hat{\theta}_n$ by solving the linear programming models.
- Use the smooth bootstrap that generates a random sample of size $n$from$\hat{\theta}_{i,} = 1, \dots, n$.
- Smooth the sampled values using the formula:

(1) $$\tilde{\theta}_i^* = \{\theta_{Bi} + h\epsilon i^* if \theta_{Bi} + h\epsilon i^* \geq 1 \; or \; 2 - \theta_{Bi} - h\epsilon i^* \; otherwise\}$$

- Use the following formula to obtain the value of $\tilde{\theta}$ by adjusting the smoothed sample value as proposed by *Farrell (1957)*.

(2) $$\theta_i^* = \bar{B} + \frac{\tilde{\theta}_i^* - \bar{B}}{(1 + \left(\frac{h^2}{\hat{\sigma}_{\theta}^2}\right))½}$$

where:

$$\bar{B} = (\frac{1}{n}) \sum_{i=1}^{n} \theta_{Bi} \; and \; \hat{\theta}_{\theta}^2 = (\frac{1}{n}) \sum_{i=1}^{n} (\hat{\theta}_i - \hat{\hat{\theta}}) \quad (3)$$

- Adjust the original input using the ratio$\hat{\theta}/\theta_i^*$.
- Resolve the original DEA model using the adjusted input to obtainθ_{kb}^*.
- Repeat steps 2 to 6 *B* times to provide for *B* sets of estimations which are; samples generated for each bank.

In equation 1 above, *h* is the smoothing parameter, and $\boldsymbol{\varepsilon}$ is a randomly drawn error term. According to Walden (2006), the *h* value is the most difficult step in the procedure. This study uses an alternative procedure of the *"normal reference rule"*, where the *h* value is calculated using the following formula:

$$h = \left[4/(p + q + 2)^{\left(\frac{1}{p}+q+4\right)} * N^{(-\frac{1}{p}+q+4)}\right] \quad (4)$$

where p equals the number of inputs, q is the number of outputs, and$N$ refers to the number of observations in the sample. The bias of the original estimate of theta will be calculated once the number of desired samples is generated. The following formula will then be computed.

$$bias\hat{\theta}_k = B^{-1} \textstyle\sum_{b=1}^{B} \quad \hat{\theta}_{kb}^* - \hat{\theta}_{kb} \quad (5)$$

Then, a bias-corrected estimator of the true value of theta, $\theta(x, y), \hat{\theta}_k^*$, can be computed using the following formula developed by Simar and Wilson (2000).

$$\hat{\theta}_k^* = \hat{\theta}_k - bias\hat{\theta}_k$$

$$= 2 * \hat{\theta} - B^1 \sum_{b=1}^{B} \hat{\theta}_{kb}^* \quad (6)$$

Given the debate about sample size in the literature, this study realized that there might be a sampling error problem due to our small sample size. Brought by the example of 21 commercial banks in the Asia-Pacific region, this small sample was within the context of not many banks using derivative instruments in their risk management operations. The deficiencies in the sample may create hurdles for more holistic analyses, so this issue will be addressed by a BDEA approach to be performed on the Performance Improvement Management (PIM-DEA) Software, which was introduced by Emrouznejad (2010) to estimate technical efficiency. This approach provides an advanced alternative that can estimate confidence intervals on DEA efficiencies and incorporate bias correction factors. Thus, this study presents the bootstrap efficiency scores for individual banks in Asia-Pacific countries.

5. Example of Comparative Findings between DEA and BDEA approach in Pure Technical Efficiency

In this study, two stages of bootstrapping were performed. In the first stage bootstrapping DEA was used for efficiency bias estimations in pure technical efficiency (PTE) of the original DEA estimates, where the bootstrapping results are compared to PTE results using the following formula:

$$PTE_1 = (PTE_0 - Bias) \quad (7)$$

Where PTE_1 denotes new pure technical efficiency, PTE_0 denotes previous pure technical efficiency derived from original DEA estimates and $Bias$ denotes the bias derived from bootstrapping analysis. In the second stage, the bias estimates found in the first stage analysis will be subtracted from PTE_0scores and times with scale efficiency (SE) scores. Therefore, the final efficiency results after bias correction are estimated. New efficiency scores are based on the following formula:

$$OTE_1 = PTE_1 x\ SE \quad (8)$$

Where OTE_1 denotes new overall technical efficiency, PTE_1 denotes new pure technical efficiency and SE denotes scale efficiency. This decomposition is unique and depicts the sources of inefficiency accurately, clarifying whether it is caused by inefficient operation PTE, disadvantageous scale efficiency SE, or both. This study presents individual banks yearly (2007-2012) which presents the efficiency of PTE and bias-corrected estimates trend.

A Simulation of Bootstrapping Results Comparing to Pure Technical Efficiency: Extracted from the findings from Zakaria and Islam (2019), in the first stage of analysis, Table 2 summarizes the simulation of findings of annual mean efficiency of 21 Asia-Pacific banks over the period 2010-2012. Column 2 lists the average (mean) of PTE from normal DEA estimated efficiency, and column 3 lists the bias-corrected by bootstrapping for each year. Column 4 presents the average amount of bias by comparing estimated efficiency and bias-corrected estimates. Although the overall mean score indicates that all banks have been inefficient over the years, the whole banking industry risk management efficiency level declined over the period 2010-2012, and declined considerably in 2011, while in 2010, the efficiency scores almost achieve perfectly efficient status. The findings, thus, modified the efficiency estimator by correcting the estimated bias from the original efficiency estimate.

Table 2: The Example of Comparative Findings of Annual Average PTE Estimates for Asia-Pacific Banks based on the Bootstrap Method

YEAR	PTE ESTIMATED EFFICIENCY	BDEA	BIAS-CORRECTED	BOOTSTRAP LOWER BOUND	BOOTSTRAP UPPER BOUND
2010	0.9996	0.9995	**0.00005**	0.9994	0.9995
2011	0.835	0.810	**0.025**	0.728	0.836
2012	0.990	0.989	**0.001**	0.987	0.990
Mean	**0.944**	**0.936**	**0.008**	**0.911**	**0.944**

Note: The sample of all efficiency scores is presented in three decimal places except in 2010 due to the close estimations, and the comparative findings were extracted from Zakaria and Islam (2019).

Based on the bootstrap estimates of 95% confidence interval in the last two columns, the results reflect the relevance of the theory of confidence interval constructed by Simar and Wilson (1998) as the mean (average) of estimated efficiency lies to the right of the estimated confidence intervals. The inverse of the average bias corrected Variable Return to Scale (VRS) efficiency score amounts to 93.6%. This indicates that based on average PTE, the banks in the Asia-Pacific region are managing their resources ineffectively under an exogenous environment. In addition, the results show that the bias in DEA scores is quite small. All bias scores are less than 0.03, which indicates that the results from DEA efficiency scores are relatively stable. However, the results from this table are relatively general. According to Arjomandi (2011), annual average pure technical efficiency (PTE) does not help in making a fair comparison between the performances of individual banks.

In accordance with the above calculation, it is important to highlight that the stochastic approach takes the missing error portion in efficiency measurement into account. The bias measurement could be upward or downward, while upward bias in estimation applied in stochastic BDEA, downward bias (error is considered in measuring efficiency score) is applied in traditional DEA. Hence, the efficiency results in bias-corrected analyses are influenced by the error portion discarded. The estimation of bias-corrected efficiency may produce lower or higher scores compared to technical efficiency in DEA. Assaf and Matawie (2010) noted that due to the upward bias in the original estimates and the bootstrap correction in the confidence interval (95%), the original estimates lie outside for every observation but close to the lower bound for the confidence interval, whereas the bias-corrected estimates for each observation lie within the confidence interval.

Concluding Remarks: The paper aims to highlight the efficiency trends analysis using both DEA and BDEA measurements. Methodologically, a non-parametric bootstrap provides greater robustness to the studies based on DEA and has the advantage of making it easy for the analysis of this study to estimate the precision metrics such as standard errors and confidence intervals for better risk management efficiency measurement that can be estimated from the sample data (Marinho and Araújo 2021; Davison 1997; Efron and Tibshirani 1994). With bias reduction estimates, new ventures on efficiency measurement based on the derivative instruments usage exhibit higher effectiveness in reducing risk effects. Other than reducing sampling error issues, this approach led to the development of the traditional DEA measurement. It proved to be a fruitful analysis method that led to a refined traditional efficiency model describing small data. Overall, this non-parametric BDEA approach allows the study to interpret the resulting risk measure estimates (Dyson and Shale 2010; Cotter and Dowd 2006) as potential estimates of risk management.

The method of breaking the analysis into non-biased reduction (DEA) and bias reduction (BDEA) has solved the risk management efficiency measurement with a realistic solution (Xue and Harker 1999; Sadjadi and Omrani 2010). The approach and interconnection to the classical efficiency measurement were easily contrasted for better problem-solving and decision making where it merged the two measurement concepts. Again, the method provided transparent, more robust and more flexible ways to ensure the viability of the method. Theoretically, this bootstrapping approach is introduced to strengthen the risk management concept and evaluation. The integrated methodology allows for a targeted analysis procedure that is more comprehensive (Casu and Molyneux 2003; Dong and Featherstone 2006). Specifically, a bootstrap strategy has a significant impact on enhancing the reliability of risk management modelling. Although the BDEA method used in this study is not new, bootstrapping was combined with DEA in ways that have not been experimented upon in previous studies.

In short, after outlining some weaknesses of classical DEA analysis, this paper has introduced the utility of bootstrap characteristics to assist in the development of a BDEA approach. Previous studies were reviewed to show that BDEA is able to solve the issues of uncertainty of noise in the data and small sample size (sampling error) under a linear programming optimization technique. This study provides a good solution through the accuracy of a prediction and estimation that is practical within perplexing environments and has reached a high-level efficient solution method that can solve real problems. By providing the example of risk management efficiency in banking sectors, the argument is that a combination of both DEA and BDEA analysis on risk management efficiency in banks is now able to appropriately incorporate two complementary approaches, hence providing a more sustainable and reliable efficiency measurement for organizations.

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